Semi-microscopic description of the proton- and neutron-induced backbending phenomena in some deformed even-even rare earth nuclei

R. Budaca[1] and A. A. Raduta[1,2]

[1] Institute of Physics and Nuclear Engineering, Bucharest POB MG6, Romania
[2] Academy of Romanian Scientists, 54 Splaiul Independentei, Bucharest 050094, Romania

Abstract

A semi-microscopic model to study the neutron- and proton-induced backbending phenomena in some deformed even-even nuclei from the rare earth region is proposed. The backbending phenomena are described by mixing four rotational bands, defined by a set of angular momentum projected states with a specific single-particle factor [1], and a model Hamiltonian describing a set of paired particles moving in a deformed mean field and interacting with a phenomenological deformed core. Due to the specific construction, the wave function acquires a complex structure which allows a quantitative description of the yrast states in the region of the two backbendings.

The model Hamiltonian

The particle-core system is studied using the following Hamiltonian [2]:

$$H = H_c + H_{sp} + H_{pair} + H_{pc}. \tag{1}$$

The core Hamiltonian H_c is a harmonic quadrupole boson operator:

$$H_c = \omega_0^b \hat{N} + \omega_1^b \hat{N}^2, \text{ where } \hat{N} = \sum_{\mu} b_{2\mu}^\dagger b_{2\mu}, \tag{2}$$

described by a phenomenological quadrupole coherent state $\psi_c = e^{d(b_{20}^\dagger - b_{20})}|0\rangle_b$ [3]. As for the single-particle Hamiltonian H_{sp}, this is a sum of two terms corresponding to neutrons and protons, each of them describing a set of particles in an intruder spherical shell model orbital $|nljm\rangle$:

$$H_{sp} = \sum_{i=\nu,\pi} (\varepsilon_{n_i l_i j_i} - \lambda_i) \sum_{m_i = all} c_{n_i l_i j_i m_i}^\dagger c_{n_i l_i j_i m_i}. \tag{3}$$

The alike nucleons interact among themselves also through a pairing force.

The particle-core interaction consists of two terms:

$$\begin{aligned} H_{pc} &= H_{qQ} + H_{J_f J_c}, \\ H_{qQ} &= -A_C \sum_{i=\nu,\pi} \sum_{\mu, m_i m_i'} \langle n_i l_i j_i m_i | r^2 Y_{2\mu} | n_i l_i j_i m_i' \rangle \\ &\quad \times c_{n_i l_i j_i m_i}^\dagger c_{n_i l_i j_i m_i'} \left[(-)^\mu b_{2-\mu}^\dagger + b_{2\mu} \right], \\ H_{J_f J_c} &= C \vec{J}_f \cdot \vec{J}_c, \text{ with } \vec{J}_f = \vec{J}_n + \vec{J}_p. \end{aligned} \tag{4}$$

The mean field is defined by averaging $\tilde{H}(= H_{sp} + H_{qQ})$ with the coherent state for the boson operators. In the first order of perturbation, its energies are given by

$$\varepsilon_{nljm} = \varepsilon_{nlj} - 4dX_C(2n+3) C^{j\,2\,j}_{\frac{1}{2}0\frac{1}{2}} C^{j\,2\,j}_{m0m}, \text{ with } X_C = \frac{\sqrt{5}\hbar A_C}{8\sqrt{\pi} M \omega_0}. \tag{5}$$

The sum of the mean field term and the pairing interaction for alike nucleons is then treated within the BCS formalism.

The particle-core space

The restricted particle-core space of angular momentum projected states with $0qp$, $2qp$ and $4qp$ is defined by:

$$\begin{aligned} \Psi_{JM}^{(1)} &= \mathcal{N}_J^{(1)} P_{M0}^J |nBCS\rangle_d |pBCS\rangle_d \psi_c \\ \Psi_{JM;1}^{(2)}(j_n\nu) &= \mathcal{N}_{J1}^{(2)}(j_n\nu) P_{M1}^J \left[J_+ \alpha_{j_n\nu}^\dagger \alpha_{j_n-\nu}^\dagger |nBCS\rangle_d \right] |pBCS\rangle_d \psi_c \\ \Psi_{JM;1}^{(3)}(j_p\pi) &= \mathcal{N}_{J1}^{(3)}(j_p\pi) P_{M1}^J |nBCS\rangle_d \left[J_+ \alpha_{j_p\pi}^\dagger \alpha_{j_p-\pi}^\dagger |pBCS\rangle_d \right] \psi_c \\ \Psi_{JM;2}^{(4)}(j_n\nu; j_p\pi) &= \mathcal{N}_{J2}^{(4)}(j_n\nu; j_p\pi) P_{M2}^J \left[J_+ \alpha_{j_n\nu}^\dagger \alpha_{j_n-\nu}^\dagger |nBCS\rangle_d \right] \\ &\quad \times \left[J_+ \alpha_{j_p\pi}^\dagger \alpha_{j_p-\pi}^\dagger |pBCS\rangle_d \right] \psi_c. \end{aligned} \tag{6}$$

When applied on a $K = 0$ pair of quasiparticles, the operator J_+ simulates the breaking of the corresponding pair which acquire a projection $K = 1$. The projection of the core coherent state is completely described within the Coherent State Model [3]. While for angular momentum projection of the many-body fermion states one used the procedure of Ref.[4] taking into account the fact that Pauli principle restrains the maximal angular momentum of a given configuration to

$$J_\tau^{max} = N_{pair}^{\tau j} \left(2j_\tau - 2N_{pair}^{\tau j} + 1 \right), \tag{7}$$

where $N_{pair}^{\tau j}$ pairs of τ particles occupy the states of angular momentum j_τ.

Band mixing

The set of projected states mentioned above is not orthogonal. Denoting by α_m^J the eigenvalues and by V_{im}^J the eigenvectors of the overlap matrix, it can be checked that the set of functions

$$\Phi_m^{JM} = \frac{1}{\sqrt{\alpha_m^J}} \sum_{i=1}^{4} \Psi_{JM}^{(i)} V_{im}^J, \; m = 1,2,3,4, \tag{8}$$

is orthogonal. The lowest eigenvalues of the total Hamiltonian H in this orthogonal basis defines the yrast band and the total wavefunction is then defined as an expansion in the same basis

$$\Phi_{Tot}^{JM} = \sum_{m=1}^{4} X_m^J \Phi_m^{JM}. \tag{9}$$

$E2$ transition operator

We suppose that the collective transition is due to the core component of the wavefunction, such that the $B(E2)$ transition probabilities are calculated by truncating the transition operator to the boson part [5],

$$Q_{2\mu} = q_1 \alpha_{2\mu} + q_2 (\alpha\alpha)_{2\mu} \tag{10}$$

where $\alpha_{2\mu}$ denotes the quadrupole collective coordinate

$$\alpha_{2\mu} = \frac{1}{\sqrt{2}} \left[b_{2\mu}^\dagger + (-)^\mu b_{2-\mu} \right]. \tag{11}$$

The microscopic structure of the yrast states have however an indirect contribution through the single-particle factor state.

Numerical results

Four rare earth even-even nuclei which present the second anomaly in the observed moments of inertia are treated within the proposed model which involves seven parameters. Six of them, namely the neutron and proton pairing constants G_n and G_p, the strengths of the qQ and spin-spin interactions, i.e. X_C and C, and the strengths ω_0^b and ω_1^b of the two boson terms, are the structure coefficients defining the model Hamiltonian. The remaining parameter d defines the coherent state ψ_c and plays the role of the deformation parameter.

- ***Energies***

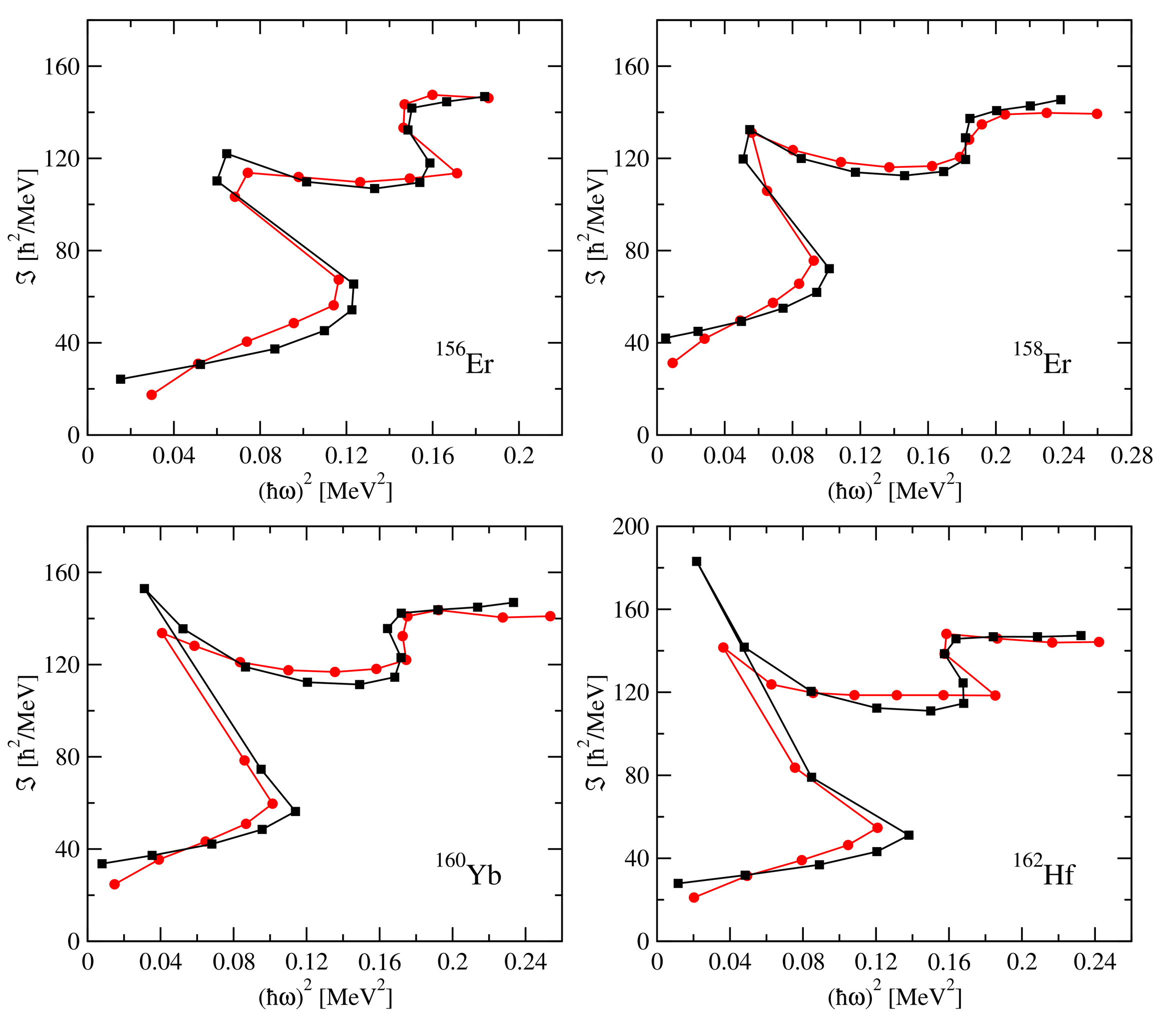
If one adopts for the moment of inertia the following expression:

$$\Im = \frac{4J+6}{E(J+2) - E(J)}, \; E(J) - \text{Yrast energies}, \tag{12}$$

and defines the rotational frequency as

$$\hbar\omega(J) = \frac{dE(J)}{dJ} \approx \frac{1}{2} \left[E(J+2) - E(J) \right], \tag{13}$$

one readily obtains the Th.(■) and Exp.(●) backbending curves.

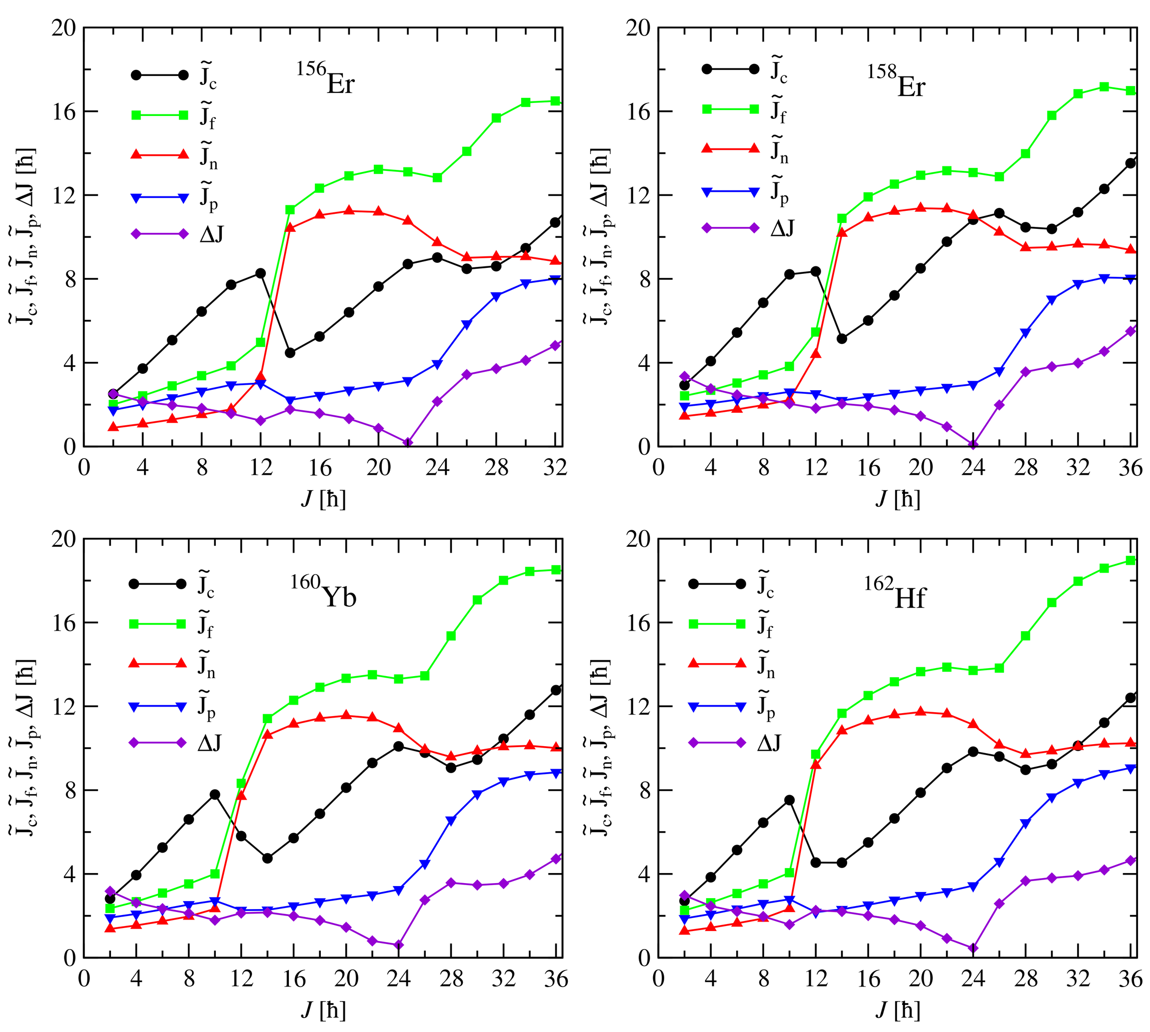


- ***Angular momentum alignment***

The averages of the involved angular momenta:

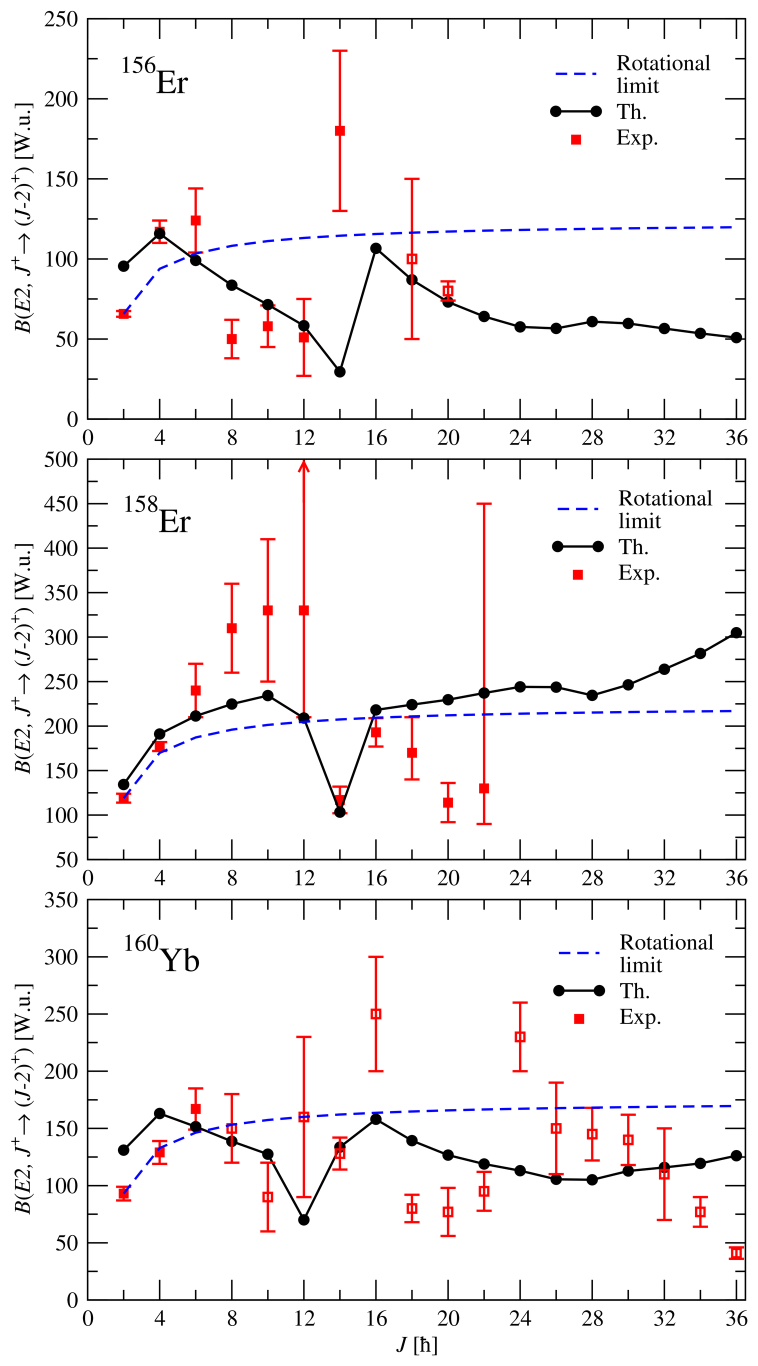
$$\tilde{J}_i(\tilde{J}_i + 1) = \langle \Phi_{Tot}^{JM} | \vec{J}_i^2 | \Phi_{Tot}^{JM} \rangle, \; i = n, p, f, c, \tag{14}$$

and the deviation $\Delta J = \left| J - (\tilde{J}_c + \tilde{J}_f) \right|$ which is a measure of the departure from the full alignment between the fermionic and core angular momenta are plotted below as functions of total angular momentum J. The dependence of these quantities on J provides a lot of information regarding the rotational dynamics of the system.



- ***Electric quadrupole transitions***

The transitions along the yrast band directly reflect the structural changes of the total wavefunction in the band crossing region. In the figure below, one compared the theoretical predictions for $B(E2)$ with experimentally available data and the rigid rotor limit.

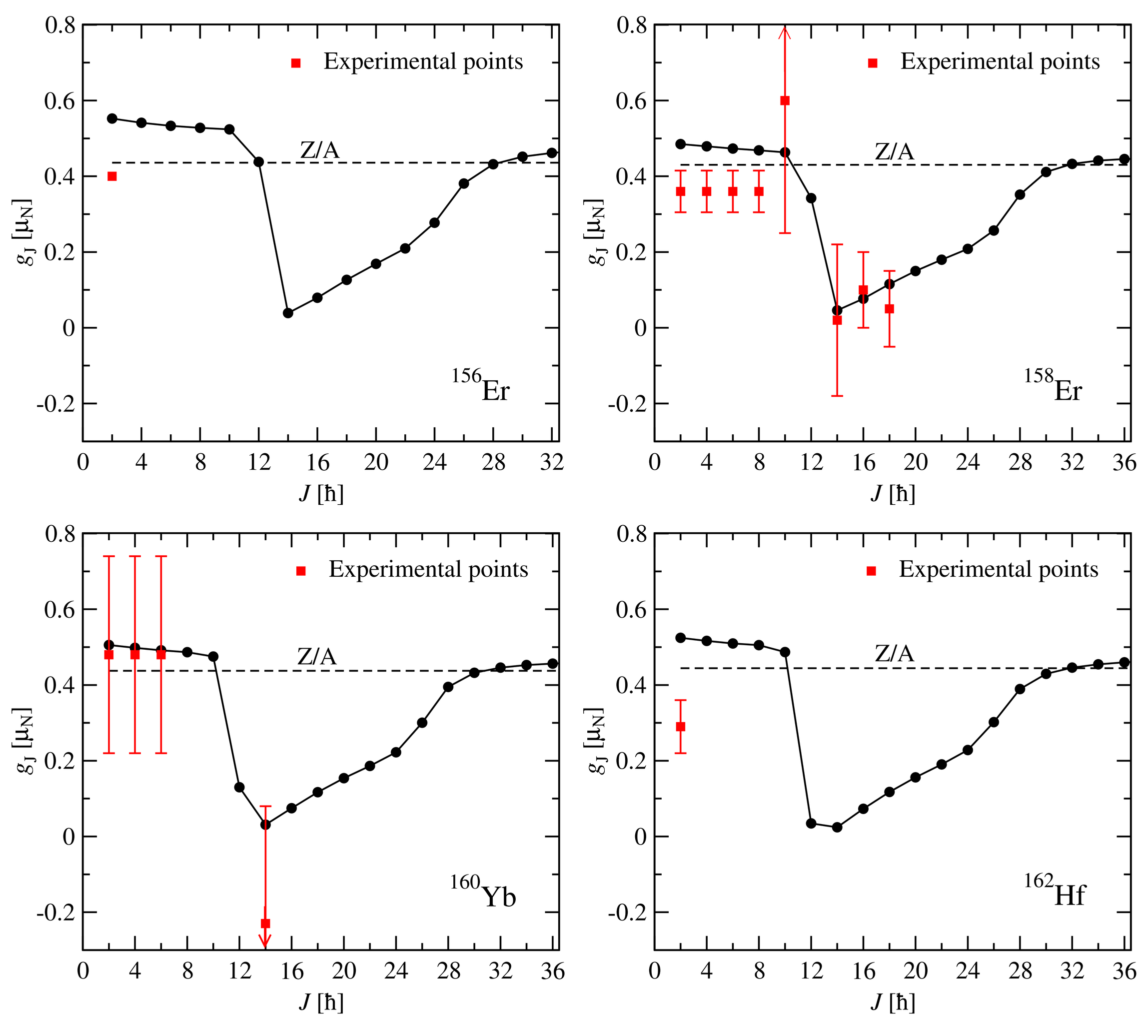


- ***Gyromagnetic factor***

The structure of the total wavefunction also dictates the behavior of the total gyromagnetic factor (figure below)

$$g_J = g_c + \frac{g_f - g_c}{2} \left[1 + \frac{\tilde{J}_f(\tilde{J}_f+1) - \tilde{J}_c(\tilde{J}_c+1)}{J(J+1)} \right]. \tag{15}$$

The fermionic g-factor is given similarly as function of the proton and neutron g-factors and average angular momenta $\tilde{J}_{n,p}$, while the core g-factor by its rotational value Z_c/A_c.



Conclusions

The present model provides a consistent explanation for the pair-breaking process in connection with the rotational alignment of the angular momenta involved in the system. Based on the energy spectrum and the electromagnetic properties of the yrast states, one identified the major differences between the neutron and the proton induced backbendings. The theoretical results suggest that the proton pair breaking is a slower process than the neutron pair breaking. Concerning the rotational alignment, it is found that the proton and neutron angular momenta first align to each other and only after that they align to the core angular momentum.

As a final conclusion, one can say that the present formalism is able to quantitatively describe the double backbending phenomenon.

References

[1] A. A. Raduta and R. Budaca, *Phys. Rev. C* **84**, 044323 (2011); **87**, 029901(E) (2013).
[2] R. Budaca and A. A. Raduta, *J. Phys. G: Nucl. Part. Phys.* **40**, 025109 (2013).
[3] A. A. Raduta, V. Ceausescu, A. Gheorghe and R. M. Dreizler, *Nucl. Phys. A* **381**, 253 (1982).
[4] A. Kelemen and R. M. Dreizler, *Z. Phys. A* **278**, 269 (1976).
[5] A. A. Raduta, V. Ceausescu and A. Gheorghe, *Nucl. Phys. A* **311**, 118 (1978).

Contact e-mail: rbudaca@theory.nipne.ro