

$2\nu\beta\beta$ decay within a higher pnQRPA approach with the gauge symmetry preserved

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Abstract. A many body Hamiltonian involving the mean field for a projected spherical single particle basis, the pairing interactions for alike nucleons, a repulsive dipole-dipole proton-neutron interaction in the particle-hole (ph) channel and an attractive dipole-pairing interaction is treated by a gauge restored and fully renormalized proton-neutron quasiparticle random phase approximation formalism. Application to the $2\nu\beta\beta$ decay rate show a good agreement with the corresponding data. The Ikeda sum rule is obeyed.

1. Introduction

The $2\nu\beta\beta$ process is interesting by its own but is also very attractive because it constitutes a test for the nuclear matrix elements (m.e.) which are used for the process of $0\nu\beta\beta$ decay. The discovery of this process may provide an answer to the fundamental question, whether neutrino is a Majorana or a Dirac particle. The subject development is described by several review papers [1, 2]. The present talk refers to the $2\nu\beta\beta$ process, which is conceived as consisting of two consecutive virtual single β^- decays. The formalism yielding closest results to the experimental data is the proton-neutron random phase approximation ($pnQRPA$) which includes the particle-hole (ph) and particle-particle (pp) as independent two body interactions. The second leg of the $2\nu\beta\beta$ process is very sensitive to changing the relative strength of the later interaction, denoted hereafter by g_{pp} . It is worth mentioning that the ph interaction is repulsive while the pp one is attractive. Consequently, there is a critical value of g_{pp} for which the first root of the $pnQRPA$ equation vanishes. Actually, this is the signal that the $pnQRPA$ approach is no longer valid. Moreover, the g_{pp} value which corresponds to a transition amplitude which agrees with the corresponding experimental data is close to the mentioned critical value. That means that the result is not stable to adding corrections to the RPA picture. An improvement for the $pnQRPA$ was achieved by one of us (AAR), in collaboration, in Refs.[3], by using a boson expansion (BE) procedure. Another procedure, proposed in Ref.[4], renormalizes the dipole two quasiparticle operators by replacing the scalar components of their commutators with their average values. Such a renormalization is, however, inconsistently achieved since the scattering operators do not participate at the renormalization process. This lack of consistency was removed in Refs.[5] where a fully renormalized $pnQRPA$ ($FRpnQRPA$) is proposed. Unfortunately, all higher pnQRPA procedures mentioned above have the common drawback of violating the Ikeda sum rule (ISR) by an amount of about 20-30% [6]. It is believed that such a violation is caused by the

gauge symmetry breaking. Consequently, a method of restoring this symmetry was formulated in Ref. [7].

Recently [8, 9], the results of Ref.[7] were improved in two respects: a) aiming at providing a unitary description of the process for the situations when the involved nuclei are spherical or deformed, here we use a projected spherical single particle basis; b) the space of proton-neutron dipole configurations is split in three subspaces, one being associated to the single β^- decay, one to the single β^+ process, and one spanned by the unphysical states. A set of *GRFRpnQRPA* (Gauge Restored and FRpnQRPA) equations is written down in the first two subspaces mentioned above, by linearizing the equations of motion of the basic transition operators corresponding to the two coupled processes.

Results are described according to the following plan. The approach is described in Section 2. Numerical applications and discussions are given in Section 3, while the final conclusions are drawn in Section 4.

2. Approximations and the model main assumptions

We suppose that the Gamow-Teller transitions dominate the Fermi ones which seems to be a reasonable hypothesis in medium and heavy nuclei. In the exact expression for the transition probability, the leptons energy is replaced by the average value: $\Delta E = mc^2 + \frac{1}{2}Q_{\beta\beta}$, where m denotes the rest mass of the emitted electron while $Q_{\beta\beta}$ the reaction heat of the process. Consequently, the half life is factorized:

$$\begin{aligned} \left[T_{1/2}^{2\nu}(0^+ \rightarrow 0^+) \right]^{-1} &= F |M_{GT}|^2, \\ M_{GT} &= \sqrt{3} \sum_m \frac{i \langle 0 | \beta^+ | m \rangle_{ii} \langle m | m' \rangle_{if} \langle m' | \beta^+ | 0 \rangle_f}{E_m + \Delta E_1}, \end{aligned} \quad (1)$$

where $\Delta E_1 = \Delta E + E_{1^+}$ and E_m are the *pnQRPA* energies. E_{1^+} denotes the experimental energy of the first 1^+ state. The GT transition operators for the single beta transitions are denoted by β^\mp . The model proposed by our group has two main ingredients:

a) The single particle basis is obtained from a deformed basis by projecting out the good angular momentum:

$$\begin{aligned} \Phi_{nlj}^{IM}(d) &= \mathcal{N}_{nlj}^I P_{MI}^I [|nljI\rangle \Psi_g] \equiv \mathcal{N}_{nlj}^I \Psi_{nlj}^{IM}(d), \\ \Psi_g &= \exp[d(b_{20}^+ - b_{20})] |0\rangle_b. \end{aligned} \quad (2)$$

The single particle energies are described by the average values of a particle-core Hamiltonian on the projected basis.

b) To describe the states involved in Eq.2 we used the following many body Hamiltonian:

$$\begin{aligned} H &= \sum_{\tau, \alpha, I, M} \frac{2}{2I+1} (\epsilon_{\tau\alpha I} - \lambda_{\tau\alpha}) c_{\tau\alpha IM}^\dagger c_{\tau\alpha IM} \\ &- \sum_{\tau, \alpha, I, I'} \frac{G_\tau}{4} P_{\tau\alpha I}^\dagger P_{\tau\alpha I'} + 2\chi \sum_{pn; p'n'; \mu} \beta_\mu^-(pn) \beta_{-\mu}^+(p'n') (-)^\mu \\ &- 2X_{dp} \sum_{pn; p'n'; \mu} \left(\beta_\mu^-(pn) \beta_{-\mu}^-(p'n') + \beta_\mu^+(pn) \beta_{-\mu}^+(p'n') \right) (-)^\mu. \end{aligned} \quad (3)$$

In the qp representation the Hamiltonian is expressed in terms of the dipole 2qp and dipole density operators:

$$\begin{aligned} A_{1\mu}^\dagger(pn) &= \sum C_{m_p m_n}^{I_p I_n 1} a_{pI_p m_p}^\dagger a_{nI_n m_n}^\dagger, \quad B_{1\mu}^\dagger(pn) = \sum C_{m_p -m_n}^{I_p I_n 1} a_{pI_p m_p}^\dagger a_{nI_n m_n} (-)^{I_n - m_n}, \\ A_{1\mu}(pn) &= \left(A_{1\mu}^\dagger(pn) \right)^\dagger, \quad B_{1\mu}(pn) = \left(B_{1\mu}^\dagger(pn) \right)^\dagger. \end{aligned} \quad (4)$$

Linearized equations of motion of the above operators determine the dipole excitations of the many body system. Such equations are obtained by the mutual commutators:

$$\begin{aligned} [A_{1\mu}(k), A_{1\mu'}^\dagger(k')] &\approx \delta_{k,k'}\delta_{\mu,\mu'} \left[1 - \frac{\hat{N}_n}{\hat{I}_n^2} - \frac{\hat{N}_p}{\hat{I}_p^2} \right], \\ [B_{1\mu}^\dagger(k), A_{1\mu'}^\dagger(k')] &\approx [B_{1\mu}^\dagger(k), A_{1\mu'}(k')] \approx 0, \\ [B_{1\mu}(k), B_{1\mu'}^\dagger(k')] &\approx \delta_{k,k'}\delta_{\mu,\mu'} \left[\frac{\hat{N}_n}{\hat{I}_n^2} - \frac{\hat{N}_p}{\hat{I}_p^2} \right], \quad k = (I_p, I_n), \end{aligned}$$

with \hat{N}_τ denoting the quasiparticle number operator of type $\tau (=p,n)$. There are three distinct approximations for these equations: 1) *pnQRPA*; 2) Standard renormalized pnQRPA [4]; 3) Fully renormalized pnQRPA [5].

Denoting by $C_{I_p, I_n}^{(1)}$ and $C_{I_p, I_n}^{(2)}$ the averages of the right hand sides, with the renormalized pnQRPA vacuum state, the renormalized operators defined as

$$\bar{A}_{1\mu}(k) = \frac{1}{\sqrt{C_k^{(1)}}} A_{1\mu}, \quad \bar{B}_{1\mu}(k) = \frac{1}{\sqrt{|C_k^{(2)}|}} B_{1\mu}, \quad (5)$$

obey boson-like commutation relations:

$$\begin{aligned} [\bar{A}_{1\mu}(k), \bar{A}_{1\mu'}^\dagger(k')] &= \delta_{k,k'}\delta_{\mu,\mu'}, \\ [\bar{B}_{1\mu}(k), \bar{B}_{1\mu'}^\dagger(k')] &= \delta_{k,k'}\delta_{\mu,\mu'} f_k, \quad f_k = \text{sign}(C_k^{(2)}). \end{aligned} \quad (6)$$

Further, these operators are used to define the phonon operator:

$$C_{1\mu}^\dagger = \sum_k \left[X(k) \bar{A}_{1\mu}^\dagger(k) + Z(k) \bar{D}_{1\mu}^\dagger(k) - Y(k) \bar{A}_{1-\mu}(k) (-)^{1-\mu} - W(k) \bar{D}_{1-\mu}(k) (-)^{1-\mu} \right], \quad (7)$$

where $\bar{D}_{1\mu}^\dagger(k)$ is equal to $\bar{B}_{1\mu'}^\dagger(k')$ or $\bar{B}_{1\mu}(k)$ depending on whether f_k is + or -. The phonon amplitudes are determined by the equations:

$$[H, C_{1\mu}^\dagger] = \omega C_{1\mu}^\dagger, \quad [C_{1\mu}, C_{1\mu'}^\dagger] = \delta_{\mu\mu'}. \quad (8)$$

Unfortunately both the renormalized and fully renormalized pnQRPA violate the *ISR* by an amount of about 20-30%. The boson expansion procedure overestimate *ISR*, while the standard renormalized pnQRPA underestimate it. In Ref.[6] we have used a boson expansion formalism on the top of a renormalized pnQRPA. The result was that the departure of our predictions from *ISR* was diminished up to about 10%.

We believe that such a deviation from the ISR is caused by the fact that the renormalized ground state is not eigenstate of the nucleon total number operator.

Indeed, the state $C_{1\mu}^\dagger|0\rangle$, where $|0\rangle$ is the vacuum state for the phonon operator defined by the *FRpnQRPA* approach, with both the *ph* and *pp* interactions included, is a superposition of components describing the neighboring nuclei $(N-1, Z+1)$, $(N+1, Z-1)$, $(N+1, Z+1)$, $(N-1, Z-1)$. The first two components conserve the total number of nucleons ($N+Z$) but violate the third component of isospin, T_3 . By contrast, the last two components violate the total number of nucleons but preserve T_3 . Actually, the last two components are those which contribute to the *ISR* violation. However, one can construct linear combinations of the basic

operators $A^\dagger, A, B^\dagger, B$ which excite the nucleus (N, Z) to the nuclei $(N-1, Z+1), (N+1, Z-1), (N+1, Z+1), (N-1, Z-1)$, respectively. These operators are:

$$\begin{aligned}\mathcal{A}_{1\mu}^\dagger(pn) &= -\left[c_p^\dagger c_n^\sim\right]_{1\mu}, \quad \mathcal{A}_{1\mu}(pn) = -\left[c_p^\dagger c_n^\sim\right]_{1\mu}^\dagger, \\ \mathbf{A}_{1\mu}^\dagger(pn) &= \left[c_p^\dagger c_n^\dagger\right]_{1\mu}, \quad \mathbf{A}_{1\mu}(pn) = \left[c_p^\dagger c_n^\dagger\right]_{1\mu}^\dagger.\end{aligned}\quad (9)$$

In terms of the new operators, the many body model Hamiltonian is:

$$\begin{aligned}H &= \sum_{\tau j m} E_{\tau j} a_{\tau j m}^\dagger a_{\tau j m} + 2\chi \sum_{pn, p'n'; \mu} \sigma_{pn; p'n'} \mathcal{A}_{1\mu}^\dagger(pn) \mathcal{A}_{1\mu}(p'n') \\ &- X_{dp} \sum_{pn; p'n'; \mu} \sigma_{pn; p'n'} (-)^{1-\mu} \left(\mathcal{A}_{1\mu}^\dagger(pn) \mathcal{A}_{1, -\mu}^\dagger(p'n') + \mathcal{A}_{1, -\mu}(p'n') \mathcal{A}_{1\mu}(pn) \right),\end{aligned}\quad (10)$$

$$\sigma_{pn; p'n'} = \frac{2}{3\hat{I}_n \hat{I}_{n'}} \langle I_p || \sigma || I_n \rangle \langle I_{p'} || \sigma || I_{n'} \rangle. \quad (11)$$

The equations of motion of the operators involved in the phonon operator are determined by the commutation relations:

$$\left[\mathcal{A}_{1\mu}(pn), \mathcal{A}_{1\mu'}^\dagger(p'n') \right] \approx \delta_{\mu, \mu'} \delta_{j_p, j_{p'}} \delta_{j_n, j_{n'}} \left[U_p^2 - U_n^2 + \frac{U_n^2 - V_n^2}{\hat{I}_n^2} \hat{N}_n - \frac{U_p^2 - V_p^2}{\hat{I}_p^2} \hat{N}_p \right]. \quad (12)$$

The quasi-boson approximation replaces the r.h. side of the above equation by its average with the *GRFRpnQRPA* vacuum state denoted by:

$$D_1(pn) = U_p^2 - U_n^2 + \frac{1}{2I_n + 1} (U_n^2 - V_n^2) \langle \hat{N}_n \rangle - \frac{1}{2I_p + 1} (U_p^2 - V_p^2) \langle \hat{N}_p \rangle. \quad (13)$$

Equations of motion show that the two *qp* energies are also renormalized:

$$E^{ren}(pn) = E_p(U_p^2 - V_p^2) + E_n(V_n^2 - U_n^2). \quad (14)$$

The space of the *pn* dipole states, \mathcal{S} , is written as a sum of three subspaces defined as:

$$\begin{aligned}\mathcal{S}_+ &= \{(p, n) | D_1(pn) > 0, E^{ren}(pn) > 0, \}, \quad \mathcal{S}_- = \{(p, n) | D_1(pn) < 0, E^{ren}(pn) < 0, \}, \\ \mathcal{S}_{sp} &= \mathcal{S} - (\mathcal{S}_+ + \mathcal{S}_-), \\ \mathcal{N}_\pm &= \dim(\mathcal{S}_\pm), \quad \mathcal{N}_{sp} = \dim(\mathcal{S}_{sp}), \quad \mathcal{N} = \mathcal{N}_+ + \mathcal{N}_- + \mathcal{N}_{sp}.\end{aligned}\quad (15)$$

In \mathcal{S}_+ one defines the renormalized operators:

$$\bar{\mathcal{A}}_{1\mu}^\dagger(pn) = \frac{1}{\sqrt{D_1(pn)}} \mathcal{A}_{1\mu}^\dagger(pn), \quad \bar{\mathcal{A}}_{1\mu}(pn) = \frac{1}{\sqrt{D_1(pn)}} \mathcal{A}_{1\mu}(pn), \quad (16)$$

while in \mathcal{S}_- the renormalized operators are:

$$\bar{\mathcal{F}}_{1\mu}^\dagger(pn) = \frac{1}{\sqrt{|D_1(pn)|}} \mathcal{A}_{1\mu}(pn), \quad \bar{\mathcal{F}}_{1\mu}(pn) = \frac{1}{\sqrt{|D_1(pn)|}} \mathcal{A}_{1\mu}^\dagger(pn). \quad (17)$$

An *pnQRPA* treatment within \mathcal{S}_{sp} would yield either vanishing or negative energies. The corresponding states are therefore spurious.

FRpnQRPA with the gauge symmetry projected defines the phonon operator as:

$$\Gamma_{1\mu}^\dagger = \sum_k \left[X(k)\bar{\mathcal{A}}_{1\mu}^\dagger(k) + Z(k)\bar{\mathcal{F}}_{1\mu}^\dagger(k) - Y(k)\bar{\mathcal{A}}_{1-\mu}(k)(-)^{1-\mu} - W(k)\bar{\mathcal{F}}_{1-\mu}(k)(-)^{1-\mu} \right], \quad (18)$$

with the amplitudes determined by the *GRFRpnQRPA* equations:

$$[H, \Gamma_{1\mu}^\dagger] = \omega\Gamma_{1\mu}^\dagger, [\Gamma_{1\mu}, \Gamma_{1\mu'}^\dagger] = \delta_{\mu,\mu'}. \quad (19)$$

In order to solve the *GRFRpnQRPA* equations we need to know $D_1(pn)$ and, therefore, the averages of the qp 's number operators, \hat{N}_p and \hat{N}_n . These are written first in particle representation and then the particle number conserving term is expressed as a linear combination of $\mathcal{A}^\dagger\mathcal{A}$ and $\mathcal{F}^\dagger\mathcal{F}$ chosen such that their commutators with $\mathcal{A}^\dagger, \mathcal{A}$ and $\mathcal{F}^\dagger, \mathcal{F}$ are preserved. The final result is:

$$\begin{aligned} \langle \hat{N}_p \rangle &= V_p^2(2I_p + 1) + 3(U_p^2 - V_p^2) \left(\sum_{\substack{n',k \\ (p,n') \in \mathcal{S}_+}} D_1(p, n') (Y_k(p, n'))^2 - \sum_{\substack{n',k \\ (p,n') \in \mathcal{S}_-}} D_1(p, n') (W_k^2) \right), \\ \langle \hat{N}_n \rangle &= V_n^2(2I_n + 1) + 3(U_n^2 - V_n^2) \left(\sum_{\substack{p',k \\ (p',n) \in \mathcal{S}_+}} D_1(p', n) (Y_k(p', n))^2 - \sum_{\substack{p',k \\ (p',n) \in \mathcal{S}_-}} D_1(p', n) (W_k^2) \right). \end{aligned}$$

GRFRpnQRPA equations, the average qp numbers and the normalization factor equations are to be simultaneously considered and solved iteratively.

3. Results of the numerical analysis and discussions

The approach presented in the previous sections was applied for the transitions of fourteen double beta emitters. The parameters defining the single particle energies are those of the spherical shell model, the deformation parameter d and the parameter k are fixed as described in Ref.[10]. The proton and neutron pairing strengths are slightly different from those from the quoted reference since the dimension of the single particle basis used in the present paper is different from that from Ref.[10]. The strength χ was taken to be:

$$\chi = \frac{5.2}{A^{0.7}} MeV. \quad (5.1)$$

This expression was obtained by fitting the positions of the GT resonances in ^{40}Ca , ^{90}Zr and ^{208}Pb [13]. The strength for the attractive pn two-body interaction was chosen such that the result for the log ft value associated to one of the single beta decay of the intermediate odd-odd nucleus, be close to the corresponding experimental data. If the experimental data are missing, the restriction refers to the existent data in the neighboring region. Since for ^{100}Mo and ^{116}Cd , experimental data for the log ft values associated to the β^\pm decays of the intermediate odd-odd nuclei ^{100}Tc and ^{116}In respectively, are available, the parameters χ and χ_1 were fixed such that the mentioned data are reproduced. For these cases, the results are compared with the data from [23] in Table 1. Let us just enumerate the results obtained with the formalism described above:

- The ISR is satisfied.
- We calculated the single β^\pm strength distributions. For some of them experimental data are available. For example, β^- strength for the transitions $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ and $^{82}\text{Se} \rightarrow ^{82}\text{Kr}$ was extracted from the reactions $^{76}\text{Ge}(p,n)^{76}\text{As}$, and $^{82}\text{Se}(p,n)^{82}\text{Br}$, respectively. The agreement of the calculated strength distribution and the corresponding experimental data is quite good. For illustration, four cases are presented in Fig. 1.

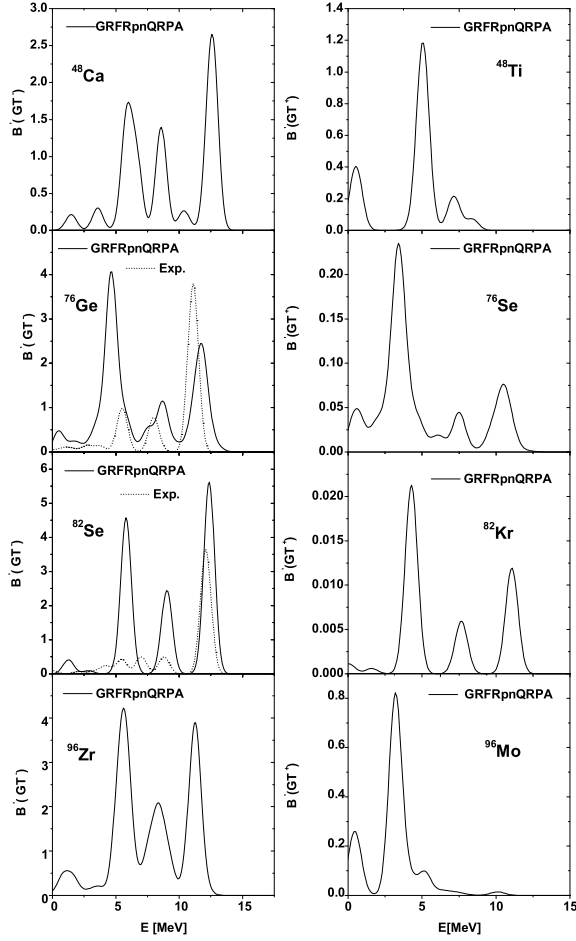


Figure 1. One third of the single β^- (left column) and one third of the β^+ (right column) strengths, denoted by $B'(GT^-)$ and $B'(GT^+)$, for the mother, ^{48}Ca , ^{76}Ge , ^{82}Se and ^{96}Zr , and daughter, ^{48}Ti , ^{76}Se , ^{82}Kr and ^{96}Mo , nuclei respectively, folded by a Gaussian function with a width of 1 MeV, are plotted as functions of the corresponding energies yielded by the present formalism. The experimental data for the β^- strengths of ^{76}Ge and ^{82}Se are also presented [14].

- Also, the summed single β^- and β^+ strengths, denoted conventionally by $\sum B(GT^-)$ and $\sum B(GT^+)$ respectively, were calculated and compared with the available experimental data. These single β decay total strengths quenched with a factor of 0.6 [12], accounting for the polarization effects on the single- β transition operator, ignored in the present paper, are listed in Table 1. Actually, the quenched values are to be compared with the experimental data, since the measured $B(GT)$ strength represents about 60%-70% of the strength corresponding to the ISR.

The experimental value for the summed $B(GT^-)$ of ^{48}Ca is taken from Ref.[15], where from the total strength, which amounts about 15.3 ± 2.2 , the contribution of isovector spin monopole states was extracted. The result was obtained with the reaction $^{48}\text{Ca}(p,n)^{48}\text{Sc}$, and corresponds to a large energy excitation interval, from 0 to 30 MeV.

In Ref.[14] the total GT strength, for ^{76}Ge and ^{82}Se , consists of the sum of the strength observed in the peaks plus the estimated contribution from the background. The experimental results correspond to 65 and 59% of the $3(N-Z)$ sum rule. According to Ref.[16], by adding to the GT cross section in discrete states the contribution from the background and that of continuum, the total strength magnitude is much improved to a better obey of the sum rule. We note a good agreement between the results of our calculations for the summed β^- strength and the corresponding experimental data.

The experimental data for the summed $B(GT^+)$ transition of ^{48}Ti , was taken from Ref.[15]. This result was obtained after extracting the contribution of the isovector spin monopole states from the total strength of 2.8 ± 0.3 . The reaction $^{48}\text{Ti}(n,p)^{48}\text{Sc}$ was used to study the

Table 1. The calculated summed strengths for the β^- strength associated to the mother nuclei and the summed β^+ strengths for the daughter nuclei, quenched by a factor 0.6, are compared with the corresponding available data. Experimental data for total $B(GT^-)$ are taken from Refs. [15] (^a), [14] (^b), [18] (^c), [21] (^d).

Nucleus	$0.6\sum B(GT)^-$	$\sum [B(GT)^-]_{exp}$
⁴⁸ Ca	14.54	14.4 ± 2.5 ^a)
⁷⁶ Ge	23.037	23.3 ^b)
⁸² Se	25.372	24.6 ^b)
⁹⁶ Zr	29.163	-
¹⁰⁴ Ru	32.921	-
¹¹⁰ Pd	32.932	-
¹²⁸ Te	43.485	40.08 ^b)
¹³⁰ Te	47.432	45.90 ^b)
¹⁴⁸ Nd	51.74	-
¹⁵⁰ Nd	54.11	-
¹⁵⁴ Sm	54.68	-
¹⁶⁰ Gd	57.93	-
Nucleus	$0.6\sum B(GT)^+$	$\sum [B(GT)^+]_{exp}$
⁴⁸ Ti	3.666	1.9 ± 0.5 ^a)
⁷⁶ Se	1.125	1.45 ± 0.07 ^c)
⁸² Kr	0.079	-
⁹⁶ Mo	2.537	0.29 ± 0.08 ^d)
¹⁰⁴ Pd	3.990	-
¹¹⁰ Cd	7.239	-
¹²⁸ Xe	2.917	-
¹³⁰ Xe	13.040	-
¹⁴⁸ Sm	1.29	-
¹⁵⁰ Sm	0.02	-
¹⁵⁴ Gd	0.54	-
¹⁶⁰ Dy	0.21	-

$B(GT^+)$ strength for excitation energies up to 30 MeV. This value for the total strength is larger than that reported by Alford *et al.*, in Ref. [17]

$$\sum B(GT^+) = 1.42 \pm 0.2. \quad (5.2)$$

where only contribution of states with excitation energies up to 15 MeV are taken into account. This comparison shows that, indeed, the $B(GT)$ strength is sensitive to the magnitude of the considered energy interval. In this context we mention the results obtained through the charge exchange reactions (³He,t) and (d,²He) on ⁴⁸Ca and ⁴⁸Ni respectively [19], for $B(GT^-)$ and $B(GT^+)$ with an excitation energy interval $E_x \leq 5$ MeV: 1.43(38), 0.45.

The GT strength from the ⁷⁶Se(n,p)⁷⁶As reaction [18] is 1.45 ± 0.07 and corresponds to an excitation energy $E_x \leq 10$ MeV. The authors used the multipole decomposition method. In Ref.[20] the $B(GT^+)$ strength was measured in a different reaction, ⁷⁶Se(d,²He)⁷⁶As, and different excitation energy interval, $E_x \leq 4$ MeV. The result reported is $\sum_{0-4MeV} B(GT^+) = 0.54 \pm 0.1$, which is smaller than that from Ref.[18]. The length of the energy intervals justifies the mentioned differences. We remark that the results

Table 2. The strengths $B(GT)$ of the single β^- transitions from the mother nuclei to the intermediate odd-odd nuclei excited in the states of the two components, GTR1 and GTR2, of the GT giant resonance are listed. The experimental [23] (Exp.) and theoretical (Th.) values for the centroid energies are also specified.

Exc. st.	^{100}Tc			
	Ex [MeV]		B(GT)	
	Exp.	Th.	Exp.	Th.
G1	13.3	11.16	23.1 ± 3.8	15.63
G2	8.0	8.05	2.9 ± 0.5	5.87
	^{116}In			
	Ex [MeV]		B(GT)	
	Exp.	Th.	Exp.	Th.
G1	14.5	12.37	25.8 ± 4.1	18.9
G2	8.9	7.87	6.6 ± 1.1	7.2

for the summed β^+ strength in ^{48}Ti and ^{76}Se are in reasonable good agreement with the corresponding experimental data.

The last strength mentioned in Table 2 refers to the daughter nucleus ^{96}Mo . Through the reaction $^{96}\text{Mo}(d, ^2\text{He})^{96}\text{Nb}$ the strength taken mainly by a single state, placed at 0.69 MeV, was measured. However, from Fig.1 we note that, indeed, there is a state at 0.69 MeV which catch a certain β^+ strength, but that strength is smaller than that distributed among the states lying in the energy interval of 1.8 to 7.5 MeV. More complete measurement through a (p, n) reaction on ^{96}Mo and an energy range of 0-10 MeV is necessary in order to make a fair comparison with the results presented here.

The quenched values of the total β^- strength of $^{128,130}\text{Te}$ are compared with the experimental data since the measured $B(GT^-)$ strength, as we already mentioned before, represents about 56% and 59% respectively, of the strength corresponding to the ISR. There are some claims [16] saying that adding the strength carried by the states from the continuum, the total $B(GT)$ strength are corrected up to 90% of the simple sum rule. We remark the good agreement between the calculated and experimental total strength. Note that if we replace the quenching factor by 0.56 for ^{128}Te and by 0.59 for ^{130}Te the results for the total strength would be 40.586 and 46.56 respectively which are closer to the experimental data. Unfortunately for the last four mother and for the last four daughter nuclei, there are no data available for the single β^- and single β^+ strengths, respectively.

- Experimental value [23] of the transition $0_i^+ \rightarrow 1^+$ m.e. describing the β^- strength of ^{100}Mo and ^{116}Cd was derived from the reactions output $^{100}\text{Mo}(^3\text{He}, t)^{100}\text{Tc}$, and $^{116}\text{Cd}(^3\text{He}, t)^{116}\text{In}$ at $\theta_t \approx 0^0$, while the m.e. $1^+ \rightarrow 0_f^+$ from the corresponding experimental log ft value. These quantities are compared with the results of our calculations in Table 2.
- Transition amplitudes and half lives were calculated for 14 double beta emitters and the results are shown in Table 3.
- The log ft values associated with the single beta transitions of the intermediate odd-odd nucleus to the daughter and mother nuclei respectively, were calculated. Results are given in Table 4.

Table 3. The Gamow-Teller transition amplitude for the $2\nu\beta\beta$ process, in units of MeV^{-1} , and the corresponding half life ($T_{1/2}$), in units of yr , are listed. The references list for experimental data is given in Ref.[25, 22].

	M_{GT}	$T_{1/2}[\text{yr}]$		
		present	Exp.	Klapdor <i>et al</i>
^{48}Ca	0.045	4.72×10^{19}	$4.2 \pm 1.2 \times 10^{19}$	3.2×10^{19}
^{76}Ge	0.177	0.938×10^{21}	$1.5 \pm 0.1 \times 10^{21}$	2.61×10^{20}
^{82}Se	0.083	1.293×10^{20}	$1.1^{+0.8}_{-0.3} \times 10^{20}$	0.85×10^{20}
^{96}Zr	0.115	1.59×10^{19}	$(1.4^{+3.5}_{-0.5}) \times 10^{19}$	5.2×10^{17}
^{100}Mo	0.221	8.79×10^{18}	$(8.0 \pm 0.16) \times 10^{18}$	2.9×10^{18}
^{104}Ru	0.453	2.26×10^{21}	-	1.8×10^{21}
^{110}Pd	0.188	3.11×10^{20}	-	1.2×10^{21}
^{116}Cd	0.160	2.02×10^{19}	$(3.2 \pm 0.3) \times 10^{19}$	5.1×10^{19}
^{128}Te	0.056	1.43×10^{24}	$(7.2 \pm 0.3) \times 10^{24}$	1.2×10^{23}
^{130}Te	0.023	1.56×10^{21}	$(1.5-2.8) \times 10^{21}$	1.9×10^{19}
^{148}Nd	0.422	2.00×10^{19}	-	1.19×10^{21}
^{150}Nd	0.042	2.50×10^{19}	$\geq 1.8 \times 10^{19}$	1.66×10^{19}
^{154}Sm	0.303	2.02×10^{21}	-	1.49×10^{22}
^{150}Gd	0.111	1.02×10^{21}	-	2.81×10^{21}

Table 4. The $\log ft$ values characterizing the β^+/EC and β^- processes associated to the intermediate odd-odd nuclei are listed.

Mother			odd-odd		Daughter
^{48}Ca	Th.	8.44	^{48}Sc	4.63	^{48}Ti
^{76}Ge	Th.	4.57	^{76}As	6.13	^{76}Se
^{82}Se	Th.	8.11	^{82}Br	7.18	^{82}Kr
^{96}Zr	Th.	5.67	^{96}Nb	7.00	^{96}Mo
^{100}Mo	Exp.	$4.45^{+0.18}_{-0.30}$	^{100}Tc	4.66	^{100}Ru
	Th.	4.65		4.1	
^{104}Ru	Exp.	4.32	^{104}Rh	4.55	^{104}Pd
	Th.	4.71		6.47	
^{110}Pd	Exp.	4.08	^{110}Ag	4.66	^{110}Cd
	Th.	4.14		6.32	
^{116}Cd	Exp.	$4.45^{+0.18}_{-0.30}$	^{116}In	4.66	^{116}Sn
	Th.	4.65		4.1	
^{128}Te	Exp.	5.049	128	6.061	^{128}Xe
	Th.	5.87		6.06	
^{130}Te	Th.	6.08	^{130}I	5.80	^{130}Xe
^{148}Nd	Th.	6.8	^{148}Pm	7.33	^{148}Sm
^{150}Nd	Th.	5.55	^{150}Pm	8.46	^{150}Sm
^{154}Sm	Th.	5.52	^{154}Eu	5.13	^{154}Gd
^{160}Gd	Th.	5.25	^{160}Tb	4.20	^{160}Dy

4. Summary and conclusions

Summarizing the results of this paper, one may say that *restoring the gauge symmetry from the fully renormalized pnQRPA provides a consistent and realistic description of the transition rate and, moreover, the ISR is obeyed.*

As shown in the quoted original paper, it seems that there is no need to include the pp interaction in the many body treatment of the process. Indeed, in the framework of a $pnQRPA$ approach this interaction violates the total number of particles and consequently the gauge projection process makes it passive. The proton-neutron correlations in the ground state are however determined by an attractive dipole pairing interaction. The results of our calculations are compared with those obtained by different methods as well as with the available experimental data. The strength of the ph interaction was taken as given by Eq.(5.1), while the one for the dipole-pairing interaction was approximately fixed such that one decay branch of the intermediate odd-odd nucleus has the $\log ft$ value close to those known for the given nuclei or for the nuclei belonging to the neighboring region. Small deviations of the predicted and experimental GT resonance centroids suggest that the parameter χ should be fixed by fitting the centroids within the $GRFRpnQRPA$. By contrast to the standard $pnQRPA$ models where the strength of the pp interaction is not affecting the position of the GT resonance centroids, here the attractive interaction contributes to the distribution of the β^- strength. Therefore, the two strengths should be fixed at a time by fitting two data, either the GT resonance centroid and the $\log ft$ value of one decay of the intermediate odd-odd nuclei or by fixing the $\log ft$ values corresponding to the single beta decays of the odd-odd intermediate nucleus.

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