The study of the shape evolution in the $^{180-196}$Pt isotope chain

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Abstract

The spectroscopic properties of even-even isotopes of Pt are studied within two solvable models For the light isotopes the sextic potential plus spheroidal function approach is used, while for the heavier ones the spheroidal plus Mathieu functions method is more suitable. From the distribution probability for the gamma coordinate one concludes on the shape evolution from prolate to oblate passing through the intermediate triaxial shape. Results for energies and B(E2) values are compared with those obtained through the Z(5) and X(5) solvable models.

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I. INTRODUCTION

Since the liquid drop model was developed [1], the quadrupole shape coordinates were widely used by both phenomenological and microscopic formalisms to describe the basic properties of nuclear systems. Based on these coordinates, one defines quadrupole boson operators in terms of which model Hamiltonians and transition operators are defined. Since the original spherical harmonic liquid drop model was able to describe only a small amount of data for spherical nuclei, several improvements have been added. Thus, the Bohr-Mottelson model was generalized by Faessler and Greiner[2] in order to describe the small oscillations around a deformed shape which results in obtaining a flexible model, called vibration rotation model, suitable for the description of deformed nuclei. Later on [3] this picture was extended by including anharmonicities as low order invariant polynomials in the quadrupole coordinates. With a suitable choice of the parameters involved in the model Hamiltonian the equipotential energy surface may exhibit several types of minima [4] like spherical, deformed prolate, deformed oblate, deformed triaxial, etc. To each equilibrium shape, specific properties for excitation energies and electromagnetic transition probabilities show up. Due to this reason, one customarily says that static values of intrinsic coordinates determine a phase for the nuclear system. A weak point of the boson description with a complex anharmonic Hamiltonian consists of the large number of the structure parameters which are to be fitted. A much smaller number of parameters is used by the coherent state model (CSM) [5] which uses a restricted collective space generated through angular momentum projection by three deformed orthogonal functions of coherent type. The model is able to describe in a realistic fashion transitional and well deformed nuclei of various shapes including states of high and very high angular momentum. Various extensions to include other degrees of freedom like isospin [6], single particle[7] or octupole [8, 9] degrees of freedom have been formulated[10].

It has been noticed that a given nuclear phase may be associated to a certain symmetry. Hence, its properties may be described with the help of the irreducible representation of the respective symmetry group. Thus, the gamma unstable nuclei can be described by the $O(6)$ symmetry [**?**], the gamma triaxial nuclei by the rigid triaxial rotor $D2$ symmetry [17], the symmetric rotor by the $SU(3)$ symmetry and the spherical vibrator by the $U(5)$ symmetry. Thus, even in the 50's, the symmetry properties have been greatly appreciated. However, a big push forward was brought by the interacting boson approximation (IBA)

[18, 19], which succeeded to describe the basic properties of a large number of nuclei in terms of the symmetries associated to a system of quadrupole (d) and monopole (s) bosons which generate a $U(6)$ algebra. The three limiting symmetries $U(5)$, $O(6)$, $SU(3)$ mentioned above, are dynamic symmetries for $U(6)$. Moreover, for each of these symmetries a specific group reduction chain provides the quantum numbers characterizing the states, which are suitable for a certain region of nuclei. Besides the virtue of unifying the group theoretical descriptions of nuclei exhibiting different symmetries, the procedure defines very simple reference pictures for the limiting cases. For nuclei lying close to the region characterized by a certain symmetry, the perturbative corrections are to be included.

In Ref. [22, 23], it has been proved that on the $U(5)-O(6)$ transition leg there exists a critical point for a second order phase transition while the $U(5)-SU(3)$ leg has a first order phase transition.

Recently, Iachello [20, 21] pointed out that these critical points correspond to distinct symmetries, namely $E(5)$ and $X(5)$, respectively. For the critical value of an ordering parameter, energies are given by the zeros of a Bessel function of half integer and irrational indices, respectively.

The description of low lying states in terms of Bessel functions was used first by Jean and Willet [16], but the interesting feature saying that this is a critical picture in a phase transition and defines a new symmetry, was indeed advanced first in Ref.[20].

The departure from the gamma unstable picture has been treated by several authors [**?**] whose contributions are reviewed by Fortunato in Ref.[36]. The difficulty in treating the gamma degree of freedom consists in the fact that this variable is coupled to the rotation variables. A full solution for the Bohr-Mottelson Hamiltonian including an explicit treatment of gamma deformation variable can be found in Refs.[13, 14]. Therein, we treated separately also the gamma unstable and the rotor Hamiltonian. A more complete study of the rotor Hamiltonian and the distinct phases associated to a tilted moving rotor is given in Ref. [15].

The treatment of the γ variable becomes even more complicated when we add to the liquid drop Hamiltonian a potential depending on β and γ at a time. To simplify the starting problem related to the inclusion of the gamma variable one uses model potentials which are sums of a beta and a gamma depending potentials. In this way the nice feature for the beta variable to be decoupled from the remaining 4 variables, specific to the harmonic liquid drop, is preserved. Further the potential in gamma is expanded either around to

$\gamma = 0$ or around $\gamma = \frac{\pi}{6}$. In the first case if only the singular term is retained one obtains the infinite square well model (ISW) described by Bessel functions in gamma. If the γ^2 term is added to this term, the Laguerre functions are the eigenstates of the approximated gamma depending Hamiltonian, which results in defining the functions characterizing the X(5) approach.

The drawback of these approximation consists in that the resulting γ depending functions are not periodic as the starting Hamiltonian is. Moreover, they are orthonormalized on unbound intervals although the underlying equation was derived under the condition of $|\gamma|$ small. Moreover, the scalar product for the space of the resulting functions is not defined based on the measure $|\sin 3\gamma| d\gamma$ as happens in the liquid drop model. Under these circumstances it happens that the approximated Hamiltonian in γ looses its hermiticity.

In some earlier publications [33**?**] we proposed a scheme where the gamma variable is described by a solvable Hamiltonian whose eigenstates are spheroidal functions which are periodic. Here we give details about the calculations and describe some new numerical applications. Moreover, the formalism was completed by treating the β variable by a Schrödinger equation associated to the Davidson's potential. Alternatively we considered the equation for a five dimensional square well potential. We have shown that of the new treatment of the gamma variable removes the drawbacks mentioned above and moreover brings a substantial improvement of the numerical analysis.

Keeping the description of the gamma variable by spheroidal functions and use a new potential for the beta variable, namely the sextic oscillator plus a centrifugal term potential, which seems to be more suitable for a realistic description of more complex spectra. We call this approach as Sextic and Spheroidal Approach (SSA). In the case of trialxial nuclei where the γ variable takes values around $\gamma_0 = \frac{\pi}{6}$, the solvable model is modified by replacing the spheroidal functions description with the Mathieu functions. The resulting model is called SMA (Spheroidal and Mathieu Approach)

In the present paper we use the SSA and SMA approaches proposed by the present authors in some previous publications, to describe the excitation energies of the lowest three rotational bands, ground, beta and gamma, as well the E2 transitions between energy levels belonging either to the same or to different bands. The results are compared with those obtained by other phenomenological formalisms.

II. A SHORT PRESENTATION OF THE USED SOLVABLE MODELS

X(5) [21], Z(5) [32], ISW [33], ISWM, SSA [34] and SMA [35] are solutions given for the Bohr-Mottelson Hamiltonian [1],

$$H=-\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{1}{\beta^2\sin3\gamma}\frac{\partial}{\partial\gamma}\sin3\gamma\frac{\partial}{\partial\gamma}-\frac{1}{4\beta^2}\sum_{k=1}^{3}\frac{\hat{Q}_k^2}{\sin^2(\gamma-\frac{2\pi}{3}k)}\right]+V(\beta,\gamma), \quad (2.1)$$

amended with a potential [16, 36]

$$V(\beta,\gamma)=V_1(\beta)+\frac{V_2(\gamma)}{\beta^2}, \quad (2.2)$$

which allows to separate the β variable from the γ variable and the three Euler angles θ_1, θ_2 and θ_3. Here, with $\hat{Q}_k$ are denoted the angular momentum projections in the intrinsic reference frame. A full separation may be achieved by expanding in power series the rotor term around $\gamma_0=0$ or $\pi/6$ minima. The resulted equations are:

$$\left[-\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{\Lambda}{\beta^2}+v_1(\beta)\right]f(\beta)=\varepsilon_\beta f(\beta), \quad (2.3)$$

$$\left[-\frac{1}{\sin3\gamma}\frac{\partial}{\partial\gamma}\sin3\gamma\frac{\partial}{\partial\gamma}-W+v_2(\gamma)\right]\phi(\gamma)=\tilde{\varepsilon}_\gamma\phi(\gamma), \quad (2.4)$$

where the following notations are used:

$$v_1(\beta)=\frac{2B}{\hbar^2}V_1(\beta),\quad v_2(\gamma)=\frac{2B}{\hbar^2}V_2(\gamma),\quad \varepsilon_\beta=\frac{2B}{\hbar^2}E_\beta,\quad \tilde{\varepsilon}_\gamma=\langle\beta^2\rangle\frac{2B}{\hbar^2}E_\gamma. \quad (2.5)$$

Λ and W are contributions coming from the rotor term and them values depend on which approximation is performed. Further, from space reasons, we will present very briefly the models used in the present paper. More details may be found in Refs. [21, 32–35].

In Eq. (2.3), an infinite square well potential is used for the X(5), Z(5), ISW and ISWM models,

$$v_1(\beta)=\begin{cases}0, & \beta\leq\beta_\omega\\ \infty, & \beta>\beta_\omega\end{cases}, \quad (2.6)$$

obtaining solutions expressed in terms of the Bessel functions of irrational order

$$f_{s,L}(\beta)=C_{s,L}\beta^{-\frac{3}{2}}J_\nu\left(\frac{x_{s,L}}{\beta_\omega}\beta\right),\quad s=1,2,3,.... \quad (2.7)$$

Here, $C_{s,L}$ is the normalization factor, while $x_{s,L}$ are the Bessel zeros.

In the case of SSA and SMA, the Eq. (2.3) is reduced to the equation of the sextic oscillator with centrifugal barrier potential [37],

$$v_1^{\pm}(\beta)=(b^2-4ac^{\pm})\beta^2+2ab\beta^4+a^2\beta^6+u_0^{\pm},\ c^{\pm}=\frac{L}{2}+\frac{5}{4}+M,\ M=0,1,2,\dots . \quad (2.8)$$

Here, $c^{\pm}$ is a constat which has two different values, one for L even and other for L odd. The Eq. (2.3) with $\Lambda = L(L+1) - 2$ and potential given by Eq. (2.8) is quasi-exactly solvable, having the solutions in the form:

$$\varphi^{(M)}_{n_\beta,L}(\beta) = N_{n_\beta,L} P^{(M)}_{n_\beta,L}(\beta^2)\beta^{L+1}e^{-\frac{a}{4}\beta^4 - \frac{b}{2}\beta^2}, \;\; n_\beta = 0,1,2,...M, \tag{2.9}$$

where $N_{n_\beta,L}$ are the normalization factor, while $P^{(M)}_{n_\beta,L}(\beta^2)$ are polynomials in x^2 of n_β order.

Concerning the Eq. (2.4), we have an oscillator potential

$$v_2(\gamma) = c\frac{1}{2}(\gamma - \gamma_0)^2, \tag{2.10}$$

which has a minimum in $\gamma = 0$, for X(5), and another minimum in $\gamma_0 = \pi/6$ for Z(5). The solutions are given as a function of the generalized Laguerre polynomials, L^m_n ,

$$\eta_{n_\gamma,K}(\gamma) = C_{n,K}\gamma^{|K/2|}e^{-(3a)\gamma^2/2}L^{|K|}_n(3a\gamma^2), \;\; n = \left(\frac{n_\gamma - |K|}{2}\right), \; a = \frac{\sqrt{c}}{3}, \tag{2.11}$$

and of Hermite polynomials H_n, respectively,

$$\eta_{\bar{n}_\gamma} = N_{\bar{n}_\gamma}H_{\bar{n}_\gamma}(b(\gamma - \pi/6))e^{-b^2(\gamma-\pi/6)/2}, \; b = \left(\frac{c\langle\beta^2\rangle}{2}\right)^{1/4}, \; \bar{n}_\gamma = 0,1,2,... \tag{2.12}$$

In both cases, in Eq. (2.4) we have $W = 0$ because of the zero approximation of the rotor term. This is not the case for the ISW, ISWM, SSA and SMA models, where a second order expansion in power series of $\sin 3\gamma$ together with a periodic potential

$$v_2(\gamma) = u_1 \cos 3\gamma + u2 \cos^2 3\gamma, \tag{2.13}$$

lead to spheroidal ($S_{m,n}$) and Mathieu ($\mathcal{M}_n$) solutions:

$$\eta(\gamma) = S_{m,n}(\cos 3\gamma; c), \;\; \eta(\gamma) = \frac{\mathcal{M}_n(3\gamma; q)}{\sqrt{|\sin 3\gamma|}},. \tag{2.14}$$

The expressions of c and q will be given in the next rows.

The advantage of the Mathieu and spheroidal functions is that are periodic, defined on a bounded interval normalized to unity with the integration measure $\sin 3\gamma$, preserving the hermiticity of the initial Hamiltonian. The previous models do not satisfy this condition.

The total energy of the system is given by the summation of the eigenvalues of the β and γ equations:

$$\varepsilon = \varepsilon_\beta + \tilde{\varepsilon}_\gamma. \tag{2.15}$$

The energy expressions of the models used in the present paper, are the following:

$$\text{X(5)}: E(s,L,n_\gamma,K)-E(1,0,0,0)=B_1(x_{s,L}^2-x_{1,0}^2)+\delta_{K,2}X,\ X=A+4C, \tag{2.16}$$

$$\text{Z(5)}: E(s,L,n_\gamma=0,R)-E(1,0,0,0)=B_1(x_{s,L,R}^2-x_{1,0,0}^2),\ B_1=\frac{1}{\beta_\omega^2}\frac{\hbar^2}{2B}, \tag{2.17}$$

$$\begin{aligned}\text{ISW}:\quad & E(s,n_\gamma,m_\gamma,L,K)=B_1x_{s,L}^2+F\left[9\lambda_{m_\gamma,n_\gamma}(c)+\frac{u_1}{2}+\frac{11}{27}D-\frac{L(L+1)}{3}\right],\\ & \lambda_{m_\gamma,n_\gamma}=\frac{1}{9}\left[\tilde{\varepsilon}_\gamma-\frac{u_1}{2}-\frac{11}{27}D+\frac{1}{3}L(L+1)\right],\ c^2=\frac{1}{9}\left(\frac{u_1}{2}+u_2-\frac{2}{27}D\right),\\ & m_\gamma=\frac{K}{2},\quad D=L(L+1)-K^2-2,\ F=\frac{1}{\langle\beta^2\rangle}\frac{\hbar^2}{2B},\end{aligned} \tag{2.18}$$

$$\begin{aligned}\text{ISWM}:\quad & E(s,n_\gamma,L,R)=B_1x_{s,L}^2+F\left[9a_{n_\gamma}(L,R)+18q(L,R)-\frac{3}{4}R^2-\frac{5}{2}\right],\\ & q=\frac{1}{36}\left(\frac{10}{9}L(L+1)-\frac{13}{12}R^2+u_1-\frac{9}{4}\right),\ a_{n_\gamma}=\frac{1}{9}\left(\tilde{\varepsilon}_\gamma+\frac{3}{4}R^2+\frac{5}{2}\right)-2q,\end{aligned} \tag{2.19}$$

$$\begin{aligned}&\text{SSA}:\\ &E(n_\beta,n_\gamma,m_\gamma,L,K)=E\left[b(2L+3)+\lambda_{n_\beta}^{(M)}+u_0^\pm\right]+F\left[9\lambda_{m_\beta,n_\gamma}(c)+\frac{u_1}{2}+\frac{11}{27}D-L(L+1)\right],\\ &\lambda_{m_\gamma,n_\gamma}=\frac{1}{9}\left[\tilde{\varepsilon}_\gamma-\frac{u_1}{2}-\frac{11}{27}D+\frac{1}{3}L(L+1)\right]+\frac{2L(L+1)}{27},\ E=\frac{\hbar^2}{2B}\end{aligned} \tag{2.20}$$

$$\begin{aligned}&\text{SMA}:\\ &E(n_\beta,n_\gamma,L,R)=E\left[4bs(L)+\lambda_{n_\beta}^{(M)}(L)+u_0^\pi\right]+F\left[9a_{n_\gamma}(L,R)+18q(L,R)-\frac{3}{4}R^2-\frac{5}{2}\right],\end{aligned} \tag{2.21}$$

where, $u_0^\pm$ are constants thet are fixed such that the potential for L odd to have the same minimum energy with the potential for L even and $\lambda_{n_\beta}^{(M)}(L)$ satisfy the equation:

$$\left[-\left(\frac{\partial^2}{\partial\beta^2}+\frac{4s-1}{\beta}\frac{\partial}{\partial\beta}\right)+2b\beta\frac{\partial}{\partial\beta}+2a\beta^2\left(\beta\frac{\partial}{\partial\beta}-2M\right)\right]P_{n_\beta,L}^{(M)}(\beta^2)=\lambda_{n_\beta}^{(M)}P_{n_\beta,L}^{(M)}(\beta^2). \tag{2.22}$$

The reduced E2 transition probabilities for ISW and SSA are determined with the following formula:

$$B(E2;L_i\to L_f)=|\langle L_i||T_2^{(E2)}||L_f\rangle|^2, \tag{2.23}$$

$$T_{2\mu}^{(E2)}=t_1\beta\left[\cos\gamma D_{\mu0}^2+\frac{\sin\gamma}{\sqrt{2}}(D_{\mu2}^2+D_{\mu,-2}^2)\right]+t_2\sqrt{\frac{2}{7}}\beta^2\left[-\cos2\gamma D_{\mu0}^2+\frac{\sin2\gamma}{\sqrt{2}}(D_{\mu2}^2+D_{\mu,-2}^2)\right], \tag{2.24}$$

where the Rose's convention [38] was used for the reduced matrix elements. For ISWM and SMA, in the expression of the transition operetor (2.24), γ is substituted with $\gamma - 2\pi/3$. The X(5) and Z(5) models keep only the zero order approximation of the first term of the transition operator (2.24).

The fitted values of the parameters involved in each model are given in Tabels I-VI.

III. NUMERICAL RESULTS

TABLE I: The parameters of the X(5) model determined in respect with the experimental data of the considered nuclei.

X(5)	^{180}Pt	^{182}Pt	^{184}Pt	^{186}Pt
B_1 [keV]	19.08	18.02	17.28	18.83
X [keV]	722.46	720.74	739.68	784.17
t	500.21[W.u.]$^{1/2}$	35.31 [eb]	419.56 [W.u.]$^{1/2}$	38[eb]

TABLE II: The same as in Table I, but for the ISW model.

ISW	^{180}Pt	^{182}Pt	^{184}Pt	^{186}Pt
B_1 [keV]	16.38	16.39	17.15	16.25
F [keV]	17.32	11.33	1.02	16.82
u_1	-0.15	-31.56	-10000	-253.8
u_2	-104.56	-163.02	-10000	6.75
t_1	614.38[W.u.]$^{1/2}$	172.01 [eb]	2461.82 [W.u.]$^{1/2}$	135.07[eb]
t_2	0.[W.u.]$^{1/2}$	708.33 [eb]	12443.5 [W.u.]$^{1/2}$	466.98[eb]

TABLE III: The same as in Table I, but for the SSA model.

SSA	^{180}Pt	^{182}Pt	^{184}Pt	^{186}Pt
E [keV]	1.04	0.81	0.62	0.84
F [keV]	3.34	5.33	6.25	3.08
a	1059.13	1686.82	3030.42	1295.83
b	135	186	256	170
u_1	-821.16	-1041.8	-302.62	1470.62
u_2	-999.99	-0.0007	-261.98	-2325.92
t_1	1750.04[W.u.]$^{1/2}$	513.17 [eb]	7820.67 [W.u.]$^{1/2}$	395.68[eb]
t_2	0.[W.u.]$^{1/2}$	7003.85 [eb]	122065 [W.u.]$^{1/2}$	4572.85[eb]

TABLE IV: The parameters of the Z(5) model determined in respect with the experimental data of the considered nuclei.

Z(5)	^{192}Pt	^{194}Pt	^{196}Pt
B_1 [keV]	29.45	32.65	31.49
t [W.u.]$^{1/2}$	23.94	18.76	20.77

TABLE V: The same as in Table IV, but for the ISWM model.

ISWM	^{192}Pt	^{194}Pt	^{196}Pt
B_1 [keV]	17.84	19.87	18.27
F [keV]	13.98	18.43	9.98
u_1	9.49	5	56.53
t_1 [W.u.]$^{1/2}$	7.85	5.92	6.76
t_2 [W.u.]$^{1/2}$	41.26	45.33	101.92

TABLE VI: The same as in Table IV, but for the SMA model.

SMA	^{192}Pt	^{194}Pt	^{196}Pt
E [keV]	2.95	2.96	0.41
F [keV]	7.87	14.68	6.46
a	616.47	732.96	28322
b	22.98	33.05	250
u_1	121.79	32.74	177.05
t_1 [W.u.]$^{1/2}$	55.10	5.92	130.17
t_2 [W.u.]$^{1/2}$	1047.89	45.33	7707.6

TABLE VII: The energy spectrum of the ground and first β and γ bands of the ^{180}Pt nucleus yielded by the X(5), ISW and SSA models are compared with the corresponding experimental data taken from Ref. [24]. The energies are given in keV units.

^{180}Pt	Exp.	X(5)	ISW	SSA
2_g^+	153	133	125	126
4_g^+	411	387	366	386
6_g^+	757	724	693	749
8_g^+	1182	1131	1093	1194
10_g^+	1674	1604	1563	1705
12_g^+	2229	2139	2100	2273
14_g^+	2842	2736	2702	2891
16_g^+	3505	3392	3369	3552
18_g^+	4253	4108	4099	4253
20_g^+	4985	4882	4892	4989
22_g^+	5729	5715	5747	5757
24_g^+	6551	6605	6663	6555
26_g^+	7434	7552	7641	7379
0_β^+	478	753	647	590
2_β^+	861	993	863	809
4_β^+	1248	1425	1258	1173
6_β^+	1650	1967	1760	1632
8_β^+		2593	2348	2164
10_β^+		3292	3013	2755
2_γ^+	677	856	857	840
3_γ^+	963	971	969	954
4_γ^+	1049	1110	1105	1101
5_γ^+	1315	1269	1263	1258
6_γ^+		1447	1440	1464
7_γ^+	1727	1642	1637	1653
8_γ^+		1854	1853	1909
9_γ^+	2198	2082	2087	2122
10_γ^+		2326	2338	2421
$r.m.s.$ [keV]		128	108	58

TABLE VIII: The same as in Tabel VII but for ^{182}Pt. The corresponding experimental data are taken from Ref. [25].

^{182}Pt	Exp.	X(5)	ISW	SSA
2_g^+	155	126	121	139
4_g^+	420	366	353	412
6_g^+	775	684	666	778
8_g^+	1206	1069	1047	1216
10_g^+	1698	1515	1492	1710
12_g^+	2242	2021	1999	2251
14_g^+	2832	2585	2568	2830
16_g^+	3461	3205	3195	3442
18_g^+	4094	3881	3882	4083
20_g^+	4729	4613	4627	4749
22_g^+	5403	5400	5430	5437
24_g^+	6127	6241	6290	6143
26_g^+	6905	7136	7208	6867
0_β^+	500	712	647	537
2_β^+	856	939	860	797
4_β^+	1240	1347	1246	1185
6_β^+	1650	1859	1734	1652
8_β^+	2118	2450	2303	2180
10_β^+		3111	2943	2755
2_γ^+	668	847	849	805
3_γ^+	943	956	955	924
4_γ^+	1034	1087	1085	1079
5_γ^+	1306	1237	1234	1236
6_γ^+	1438	1405	1402	1446
7_γ^+	1731	1589	1587	1630
8_γ^+		1790	1789	1886
9_γ^+		2005	2008	2088
10_γ^+		2236	2243	2382
r.m.s. [keV]		164	156	47

TABLE IX: The same as in Tabel I but for ^{184}Pt. The corresponding experimental data are taken from Ref. [26].

^{184}Pt	Exp.	X(5)	ISW	SSA
2_g^+	163	121	120	131
4_g^+	436	351	350	393
6_g^+	798	656	654	749
8_g^+	1231	1025	1023	1176
10_g^+	1707	1453	1450	1658
12_g^+	2204	1938	1935	2185
14_g^+	2727	2478	2476	2749
16_g^+	3282	3073	3071	3344
18_g^+	3869	3721	3719	3967
20_g^+	4493	4422	4422	4611
22_g^+	5167	5176	5177	5276
24_g^+	5897	5983	5984	5957
26_g^+	6686	6841	6844	6652
28_g^+	7535	7751	7755	7360
0_β^+	492	682	677	581
2_β^+	844	900	894	822
4_β^+	1234	1291	1283	1198
6_β^+	1800	1782	1772	1655
8_β^+		2348	2336	2173
10_β^+		2982	2968	2738
2_γ^+	649	860	860	817
3_γ^+	940	965	964	932
4_γ^+	1028	1090	1090	1080
5_γ^+	1307	1235	1233	1234
6_γ^+	1463	1396	1394	1438
7_γ^+	1731	1572	1571	1617
8_γ^+		1764	1763	1866
9_γ^+		1971	1969	2064
10_γ^+		2192	2191	2351
r.m.s. [keV]		151	152	83

TABLE X: The same as in Tabel VII but for ^{186}Pt. The corresponding experimental data are taken from Ref. [27].

^{186}Pt	Exp.	X(5)	ISW	SSA
2_g^+	192	132	123	146
4_g^+	490	382	362	426
6_g^+	878	715	685	801
8_g^+	1343	1117	1080	1250
10_g^+	1858	1583	1543	1757
12_g^+	2336	2111	2073	2315
14_g^+	2825	2700	2667	2916
16_g^+	3395	3348	3325	3557
18_g^+	4051	4054	4045	4229
20_g^+	4788	4819	4827	4933
22_g^+	5597	5640	5671	5666
24_g^+	6464	6519	6575	6424
26_g^+	7408	7454	7540	7205
0_β^+	472	743	642	472
2_β^+	798	981	856	743
4_β^+	1223	1407	1247	1134
6_β^+	1600	1941	1744	1604
8_β^+		2559	2325	2135
10_β^+		3249	2982	2718
2_γ^+	607	916	917	849
3_γ^+	957	1029	1027	970
4_γ^+	992	1166	1161	1130
5_γ^+	1363	1323	1317	1290
6_γ^+	1470	1499	1492	1505
7_γ^+	1801	1691	1687	1693
8_γ^+	2004	1901	1899	1954
9_γ^+	2280	2126	2129	2163
10_γ^+	2545	2367	2377	2462
r.m.s. [keV]		166	155	107

TABLE XI: The energy spectrum of the ground and first β and γ bands of the ^{192}Pt nucleus yielded by the Z(5), ISWM and SMA models are compared with the corresponding experimental data taken from Ref. [29]. The energies are given in keV units.

^{192}Pt	Exp.	Z(5)	ISWM	SMA
2_g^+	317	297	303	214
4_g^+	785	698	772	647
6_g^+	1365	1184	1346	1247
8_g^+	2018	1747	2010	1979
10_g^+	2729	2383	2759	2820
0_β^+	1195	1163	705	1108
2_β^+	1439	1693	1254	1489
4_β^+		2366	2063	2117
6_β^+		3140	3005	2903
8_β^+		4003	4055	3807
2_γ^+	612	546	552	668
3_γ^+	921	772	798	877
4_γ^+	1201	1313	1201	1184
5_γ^+	1482	1377	1418	1418
6_γ^+	1869	2099	1953	1865
7_γ^+	2113	2041	2134	2106
8_γ^+	2591	2931	2792	2678
9_γ^+		2769	2938	2914
10_γ^+		3819	3371	3597
$r.m.s.$ [keV]		193	158	76

TABLE XII: The same as in Tabel XI but for ^{194}Pt isotope. The experimental data taken from Ref. [30]. The energies are given in keV units.

^{194}Pt	Exp.	Z(5)	ISWM	SMA
2_g^+	328	329	334	252
4_g^+	811	774	835	723
6_g^+	1412	1313	1435	1347
8_g^+	2100	1936	2120	2081
10_g^+	2848	2642	2883	2899
0_β^+	1267	1289	785	1150
2_β^+	1512	1877	1392	1623
4_β^+		2623	2272	2328
6_β^+		3482	3283	3165
8_β^+		4438	4398	4094
2_γ^+	622	605	638	627
3_γ^+	923	856	909	868
4_γ^+	1229	1456	1378	1284
5_γ^+	1499	1527	1590	1492
6_γ^+		2327	2214	2090
7_γ^+		2263	2360	2246
8_γ^+		3250	3130	3003
9_γ^+		3070	3211	3095
10_γ^+		4234	3665	3995
$r.m.s.$ [keV]		157	148	69

TABLE XIII: The same as in Tabel XI but for ^{196}Pt isotope. The experimental data taken from Ref. [31]. The energies are given in keV units.

^{196}Pt	Exp.	Z(5)	ISWM	SMA
2_g^+	356	318	314	255
4_g^+	877	747	824	748
6_g^+	1526	1266	1466	1414
8_g^+	2253	1868	2227	2215
10_g^+	3044	2548	3101	3125
0_β^+	1135	1244	722	948
2_β^+	1362	1810	1288	1428
4_β^+		2530	2145	2138
6_β^+		3358	3165	2996
8_β^+		4280	4322	3969
2_γ^+	689	584	660	724
3_γ^+	1015	825	915	951
4_γ^+	1293	1405	1290	1280
5_γ^+	1610	1473	1560	1543
6_γ^+	2007	2245	2051	2008
7_γ^+		2183	2328	2286
8_γ^+	2750	3134	2928	2870
9_γ^+		2961	3211	3151
10_γ^+		4084	3694	3841
$r.m.s.$ [keV]		274	135	92

TABLE XIV: The reduced E2 transition probabilities determined with the X(5), ISW and SSA models for the ^{180}Pt nucleus are compared with the corresponding experimental data taken from Ref. [24].

B(E2)(W.u.)	Exp.	X(5)	ISW	SSA
$2_g^+ \rightarrow 0_g^+$	153^{+15}_{-15}	106	106	110
$4_g^+ \rightarrow 2_g^+$	140^{+30}_{-30}	169	169	168
$6_g^+ \rightarrow 4_g^+$	≥ 50	210	210	202
$8_g^+ \rightarrow 6_g^+$		241	241	230
$10_g^+ \rightarrow 8_g^+$		266	265	255
$12_g^+ \rightarrow 10_g^+$		286	285	278
$14_g^+ \rightarrow 12_g^+$		302	301	300
$r.m.s.$ (W.u.)		39	39	36

TABLE XV: The reduced E2 transition probabilities determined with the X(5), ISW and SSA models for the ^{182}Pt nucleus are compared with the corresponding experimental data taken from Ref. [28].

B(E2)((eb)^2)	Exp.	X(5)	ISW	SSA
$2_g^+ \rightarrow 0_g^+$	0.66	0.53	1.02	1.02
$4_g^+ \rightarrow 2_g^+$	1.15	0.84	1.36	1.38
$6_g^+ \rightarrow 4_g^+$	1.74	1.05	1.37	1.42
$8_g^+ \rightarrow 6_g^+$	1.55	1.20	1.32	1.36
$10_g^+ \rightarrow 8_g^+$	1.63	1.32	1.24	1.25
$12_g^+ \rightarrow 10_g^+$	0.97	1.42	1.16	1.13
$14_g^+ \rightarrow 12_g^+$	0.69	1.51	1.088	1.00
$r.m.s.$ ((eb)^2)		0.49	0.32	0.29

TABLE XVI: The same as in Tabel XI but for ^{184}Pt isotope. The experimental data are taken from Ref. [26].

B(E2)(W.u.)	Exp.	X(5)	ISW	SSA
$2_g^+ \rightarrow 0_g^+$	127_{-5}^{+5}	75	179	176
$4_g^+ \rightarrow 2_g^+$	210_{-8}^{+8}	119	236	238
$6_g^+ \rightarrow 4_g^+$	226_{-12}^{+12}	148	236	243
$8_g^+ \rightarrow 6_g^+$	271_{-18}^{+18}	170	222	232
$10_g^+ \rightarrow 8_g^+$	310_{-10}^{+10}	187	205	214
$12_g^+ \rightarrow 10_g^+$	183_{-17}^{+17}	201	188	193
$14_g^+ \rightarrow 12_g^+$	165_{-17}^{+17}	213	173	171
$16_g^+ \rightarrow 14_g^+$	143_{-17}^{+17}	223	159	150
$18_g^+ \rightarrow 16_g^+$	80_{-5}^{+5}	231	147	129
$2_\beta^+ \rightarrow 0_\beta^+$	>0.095	59	141	135
$4_\beta^+ \rightarrow 2_\beta^+$	>0.13	89	201	190
$2_\beta^+ \rightarrow 0_g^+$	>0.010	1.58	11.56	6.72
$4_\beta^+ \rightarrow 2_\gamma^+$	0.0062	0.0062	7.30	1.89
$r.m.s.$ (W.u.)		86	49	43

TABLE XVII: B(E2) values for ^{186}Pt. Datare from Ref.[28].

B(E2)$(eb)^2$	Exp.	X(5)	ISW	SSA
$2_g^+ \rightarrow 0_g^+$	$0.69_{-0.05}^{+0.05}$	0.61	0.99	0.99
$4_g^+ \rightarrow 2_g^+$	$1.15_{-0.08}^{+0.08}$	0.98	1.40	1.42
$6_g^+ \rightarrow 4_g^+$	$1.77_{-0.14}^{+0.14}$	1.22	1.52	1.55
$8_g^+ \rightarrow 6_g^+$	$1.80_{-0.18}^{+0.18}$	1.40	1.55	1.59
$10_g^+ \rightarrow 8_g^+$	$1.86_{-0.16}^{+0.16}$	1.54	1.55	1.58
$12_g^+ \rightarrow 10_g^+$	$1.56_{-0.16}^{+0.16}$	1.66	1.54	1.54
$14_g^+ \rightarrow 12_g^+$	$1.38_{-0.13}^{+0.13}$	1.75	1.52	1.49
$16_g^+ \rightarrow 14_g^+$	$1.23_{-0.22}^{+0.22}$	1.84	1.51	1.42
$r.m.s.$ $(eb)^2$		0.37	0.24	0.22

TABLE XVIII: The reduced E2 transition probabilities determined with the Z(5), ISWM and SMA models for the ^{192}Pt nucleus are compared with the corresponding experimental data taken from Ref. [29].

B(E2)[W.u.]	Exp.	Z(5)	ISWM	SMA
$2_g^+ \to 0_g^+$	$57.2^{+1.2}_{-1.2}$	42.5	18.2	49.4
$4_g^+ \to 2_g^+$	89^{+5}_{-5}	68	53	73
$6_g^+ \to 4_g^+$	70^{+30}_{-30}	94	111	98
$3_\gamma^+ \to 2_\gamma^+$	102^{+10}_{-10}	92	76	89
$2_\gamma^+ \to 0_g^+$	$0.55^{+0.04}_{-0.04}$	0	0.12	0.93
$3_\gamma^+ \to 2_g^+$	$0.68^{+0.07}_{-0.07}$	0	0.54	1.74
$3_\gamma^+ \to 4_g^+$	38^{+10}_{-10}	53	38	38
r.m.s. [W.u.]		15	27	14

B(E2)[W.u.]	Exp.	Z(5)	ISWM	SMA
$2_g^+ \to 0_g^+$	$49.2^{+0.8}_{-0.8}$	26.1	5.8	24.8
$4_g^+ \to 2_g^+$	85^{+5}_{-5}	41	17	37
$6_g^+ \to 4_g^+$	67^{+21}_{-21}	57	36	51
$8_g^+ \to 6_g^+$	50^{+14}_{-14}	69	53	61
$10_g^+ \to 8_g^+$	34^{+9}_{-9}	77	68	70
$0_\beta^+ \to 2_g^+$	$0.63^{+0.14}_{-0.14}$	19.55	6.85	9.13
$0_\beta^+ \to 2_\gamma^+$	$8.4^{+1.9}_{-1.9}$	0	0.05	39.9
$2_\gamma^+ \to 0_g^+$	$0.29^{+0.04}_{-0.04}$	0	0.05	1.29
$2_\gamma^+ \to 2_g^+$	89^{+11}_{-11}	42	90	71
$4_\gamma^+ \to 2_\gamma^+$	21^{+4}_{-4}	19	3	15
$4_\gamma^+ \to 2_g^+$	$0.36^{+0.07}_{-0.07}$	0	0.02	0.79
$4_\gamma^+ \to 4_g^+$	14	9	5	15.5
$r.m.s.$ [W.u.]		25	28	22

TABLE XIX: The same as in Tabel XVI but for ^{194}Pt. Data are from [30].

TABLE XX: The same as in Tabel XVI but for ^{196}Pt. The corresponding experimental data are taken from Ref. [31].

B(E2)[W.u.]	Exp.	Z(5)	ISWM	SMA
$2_g^+ \to 0_g^+$	$40.6^{+0.2}_{-0.2}$	32	11	34.1
$4_g^+ \to 2_g^+$	$60^{+0.9}_{-0.9}$	51	32	52
$6_g^+ \to 4_g^+$	73^{+4}_{-73}	70	67	72
$8_g^+ \to 6_g^+$	78^{+10}_{-78}	84	99	87
$2_\beta^+ \to 0_\beta^+$	5^{+5}_{-5}	25	5.5	23
$0_\beta^+ \to 2_g^+$	$2.8^{+1.5}_{-1.5}$	24	12.9	15.5
$0_\beta^+ \to 2_\gamma^+$	18^{+10}_{-10}	0	0	21
$2_\beta^+ \to 0_g^+$	$0.0025^{+0.0024}_{-0.0024}$	0.344	0	0.498
$2_\beta^+ \to 2_\gamma^+$	$0.26^{+0.23}_{-0.23}$	4.61	7.81	0.02
$2_\beta^+ \to 4_g^+$	$0.13^{+0.12}_{-0.12}$	10.47	2.52	9.83
$4_\gamma^+ \to 2_\gamma^+$	29^{+6}_{-29}	24	5	21
$4_\gamma^+ \to 2_g^+$	$0.56^{+0.12}_{-0.17}$	0	0.01	0.32
$6_\gamma^+ \to 4_\gamma^+$	49^{+13}_{-13}	33	10	29
$6_\gamma^+ \to 4_g^+$	$0.48^{+0.14}_{-0.14}$	0	0	0.19
$6_\gamma^+ \to 6_g^+$	16^{+5}_{-5}	6	10	5
$r.m.s.$ [W.u.]		11	18	9

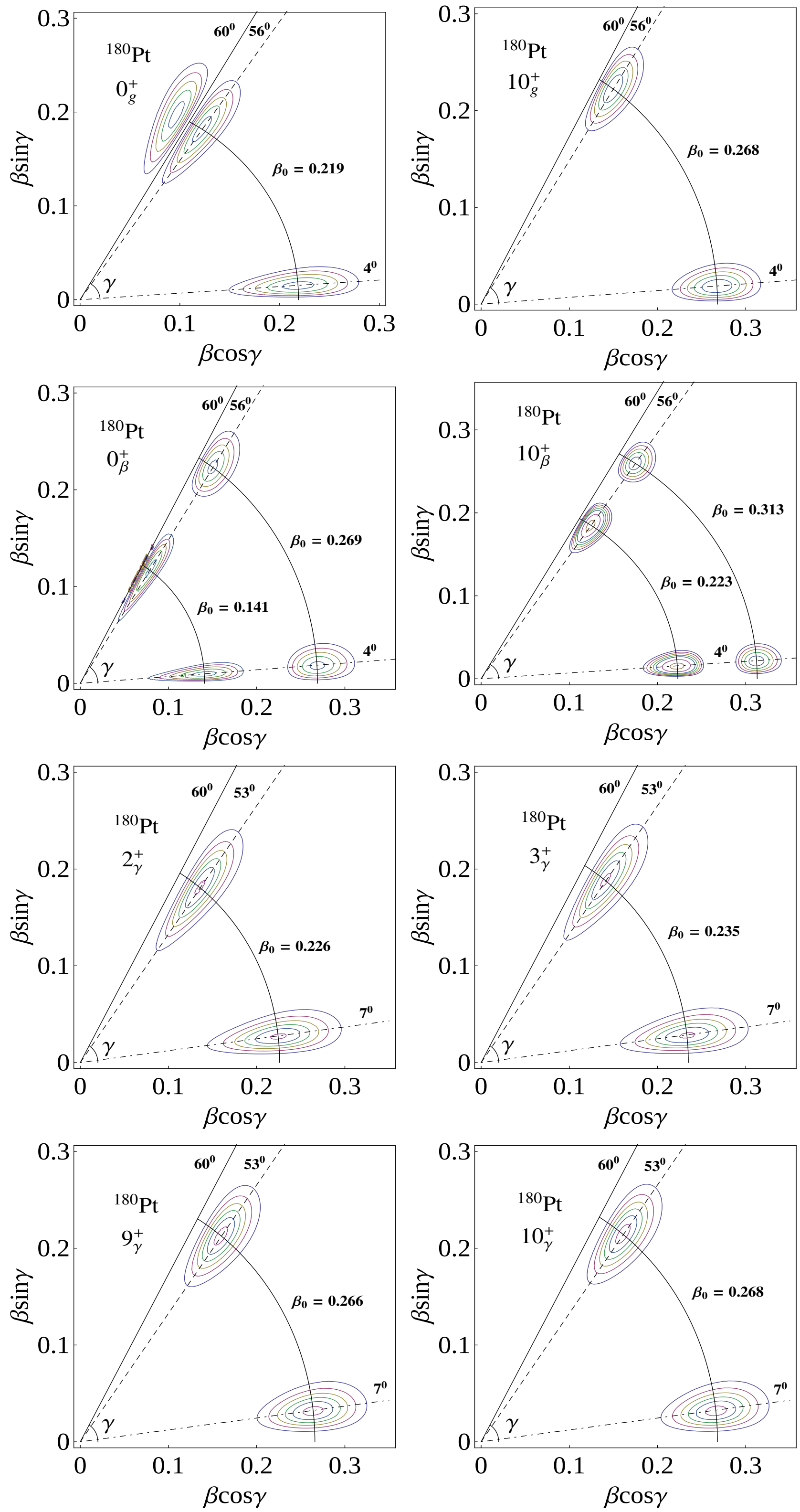

FIG. 1: Density probabilities for the 0_g^+, 10_g^+, 0_β, 10_β, 2_γ^+, 3_γ^+, 9_γ^+ and 10_γ^+ states of the ^{180}Pt isotope determined usind SSA.

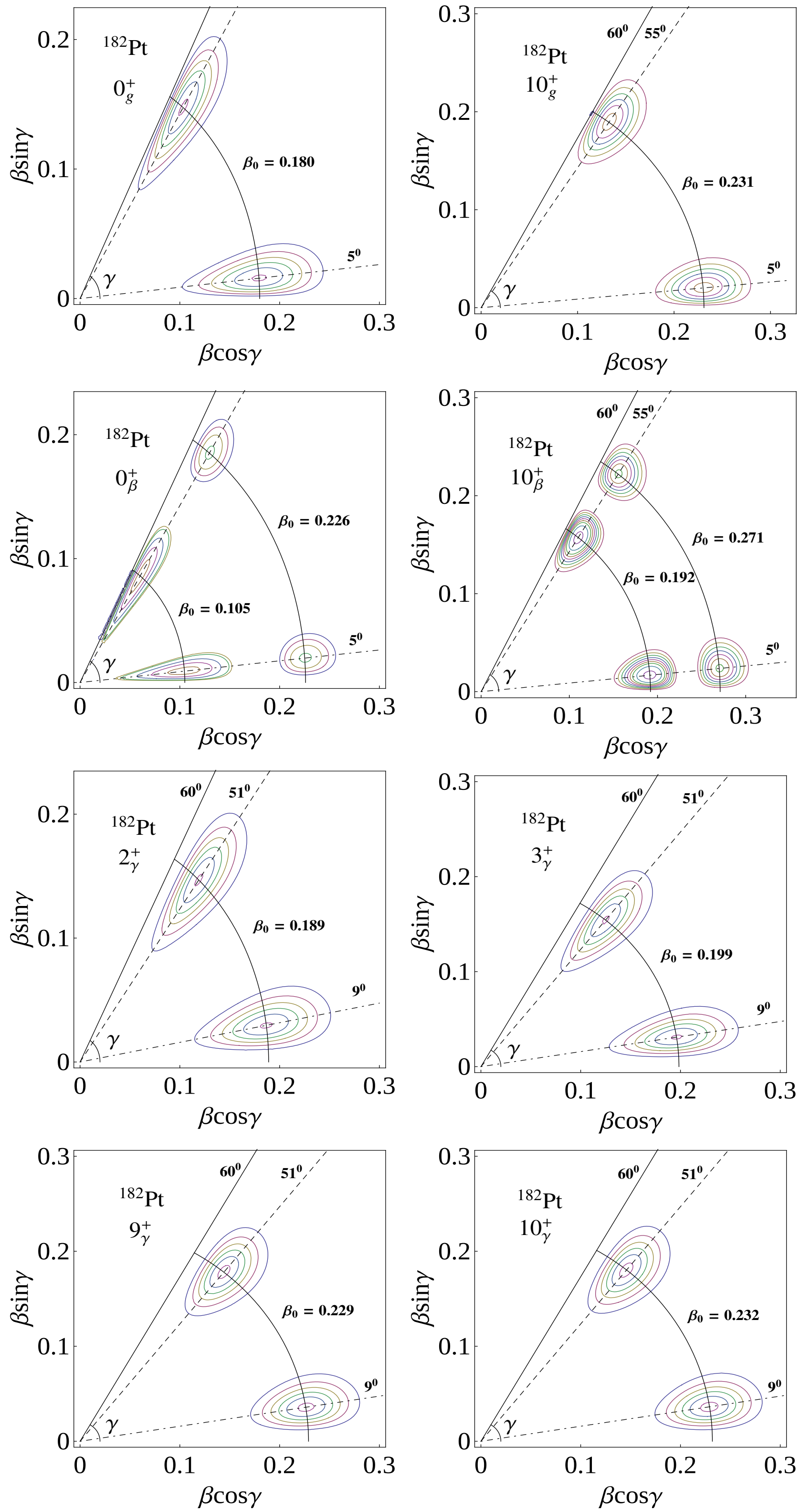

FIG. 2: The same as in Fig. 2 but for ^{182}Pt.

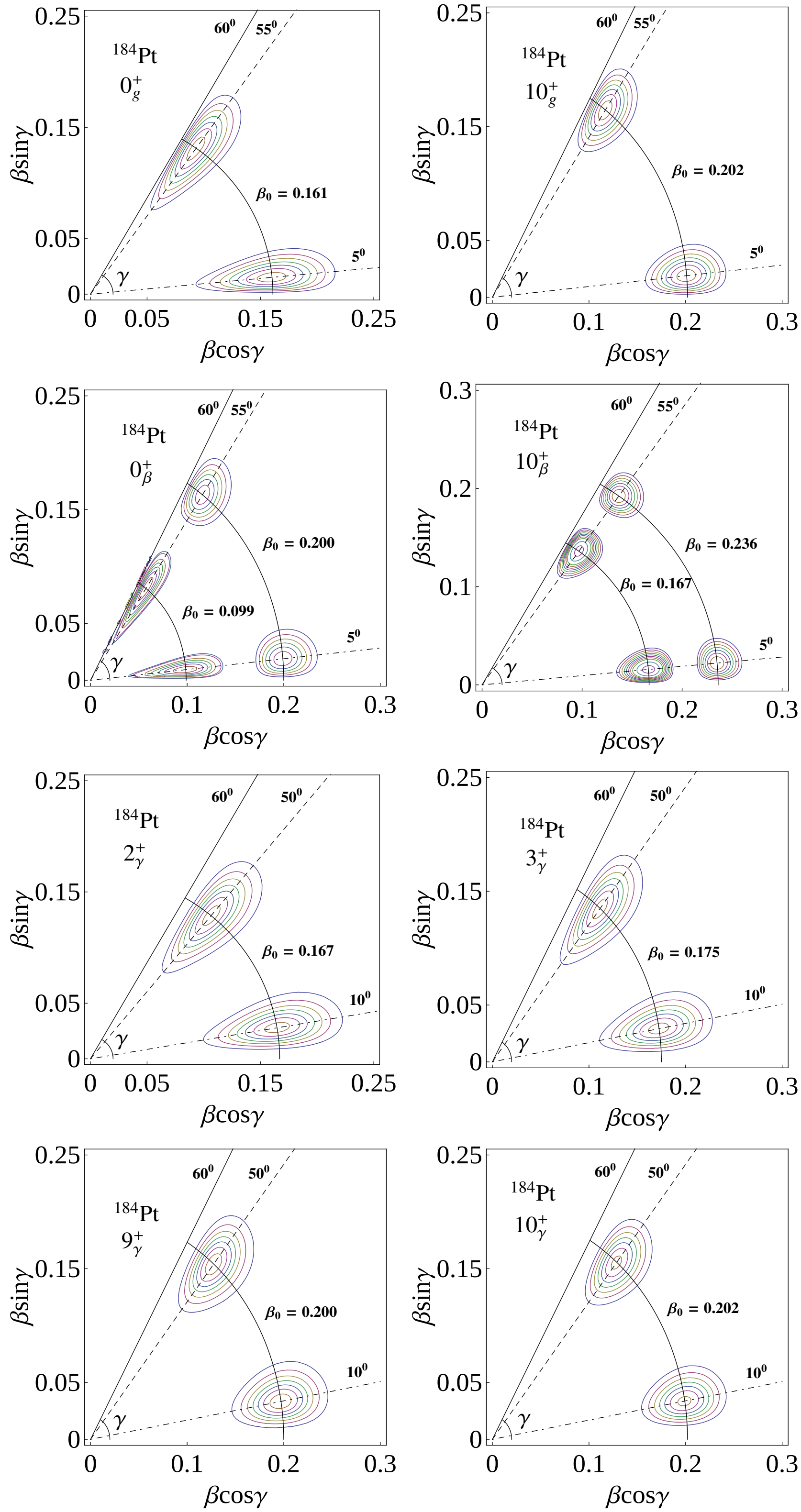

FIG. 3: The same as in Fig. 2 but for ^{184}Pt.

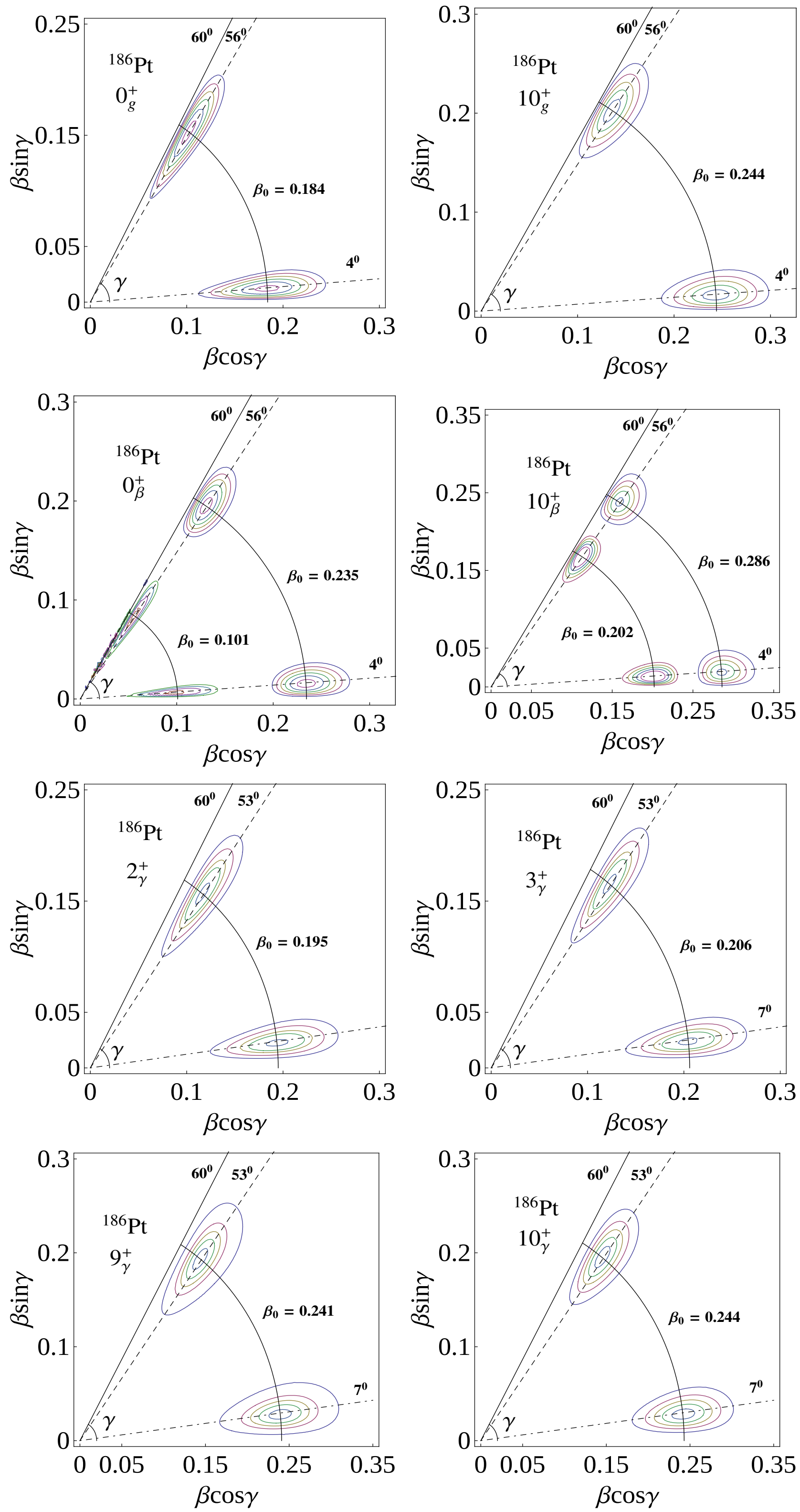

FIG. 4: The same as in Fig. 2 but for ^{186}Pt.

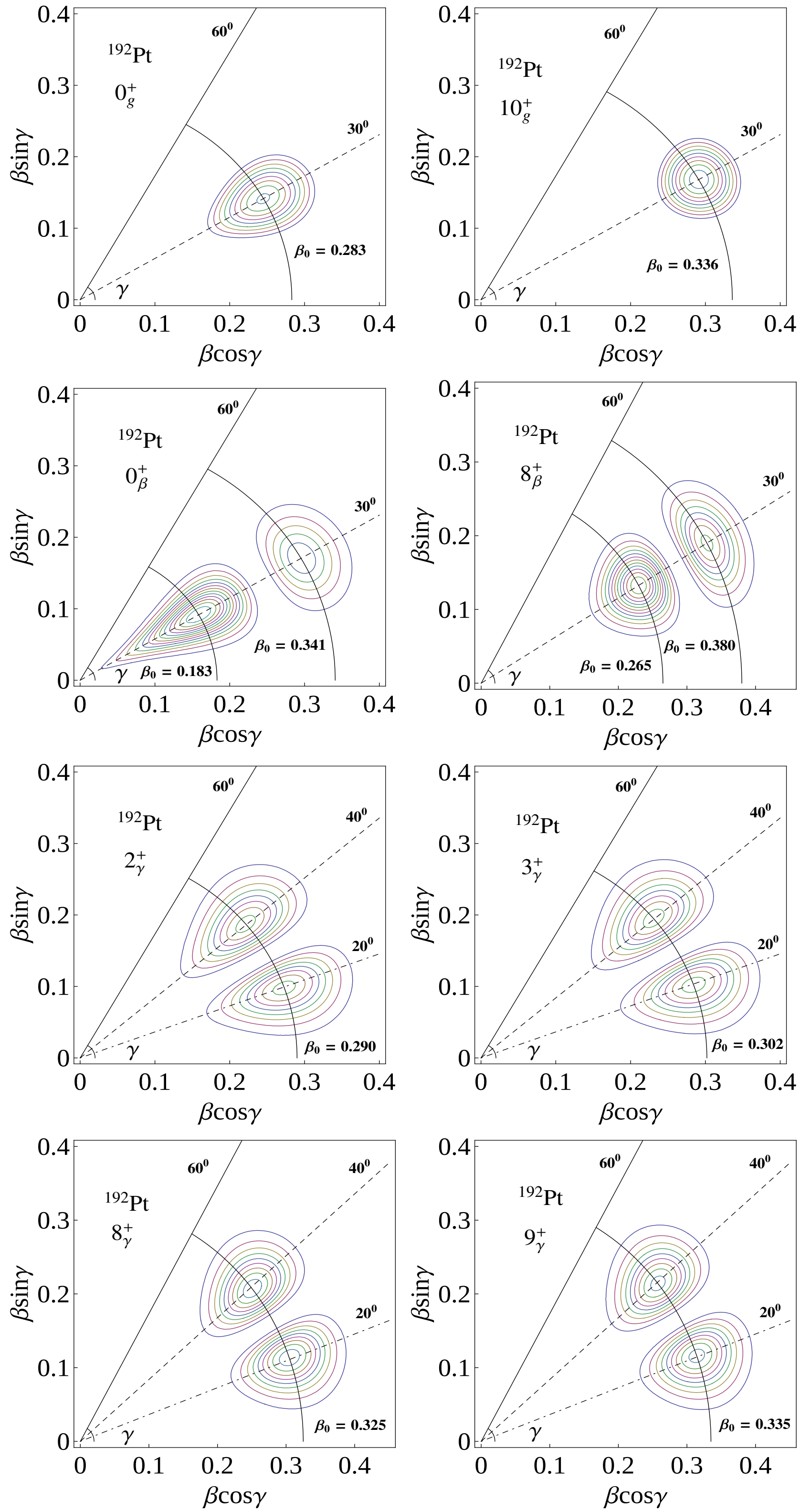

FIG. 5: The same as in Fig. 2 but for ^{192}Pt.

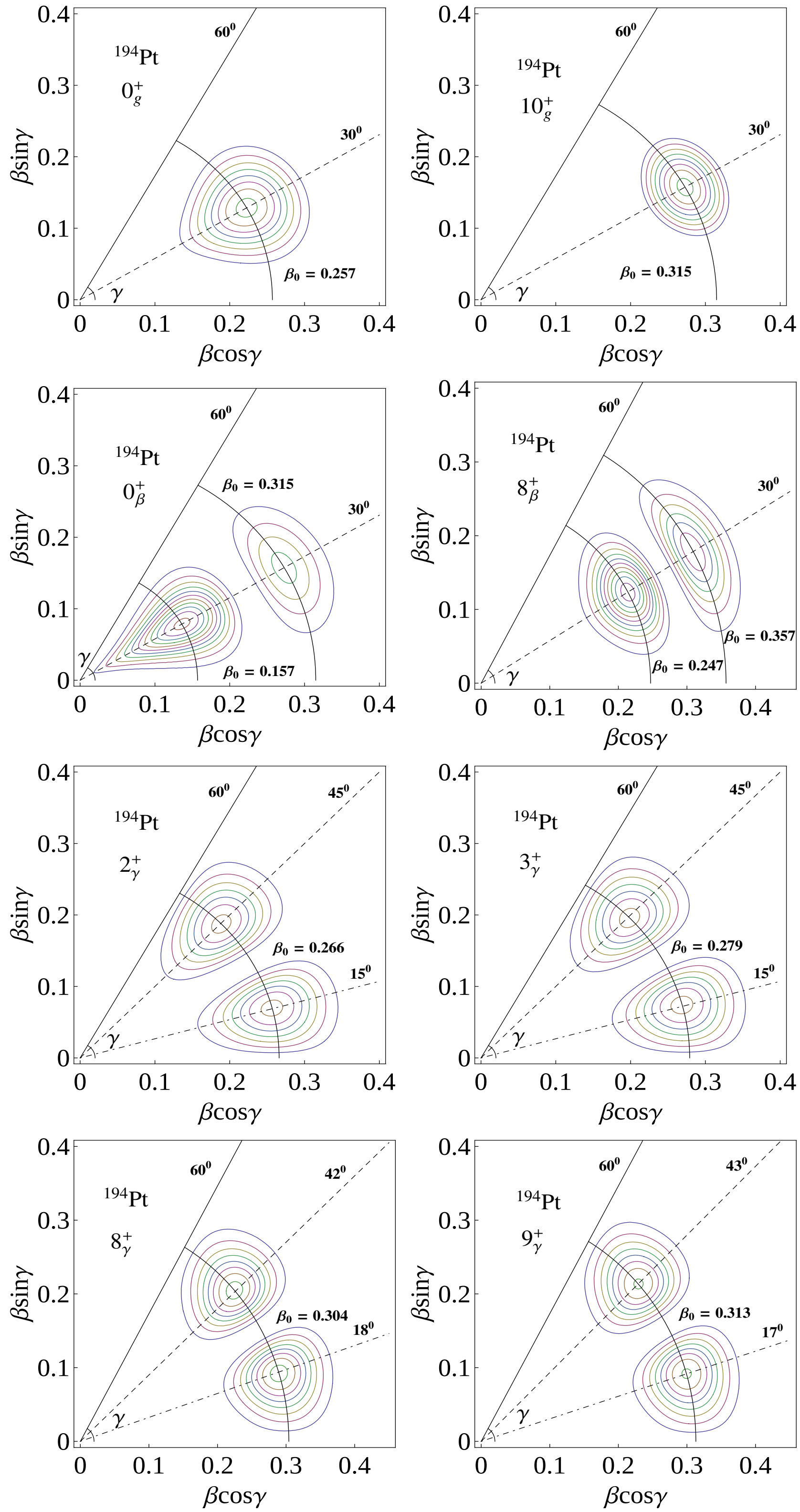

FIG. 6: The same as in Fig. 2 but for ^{194}Pt.

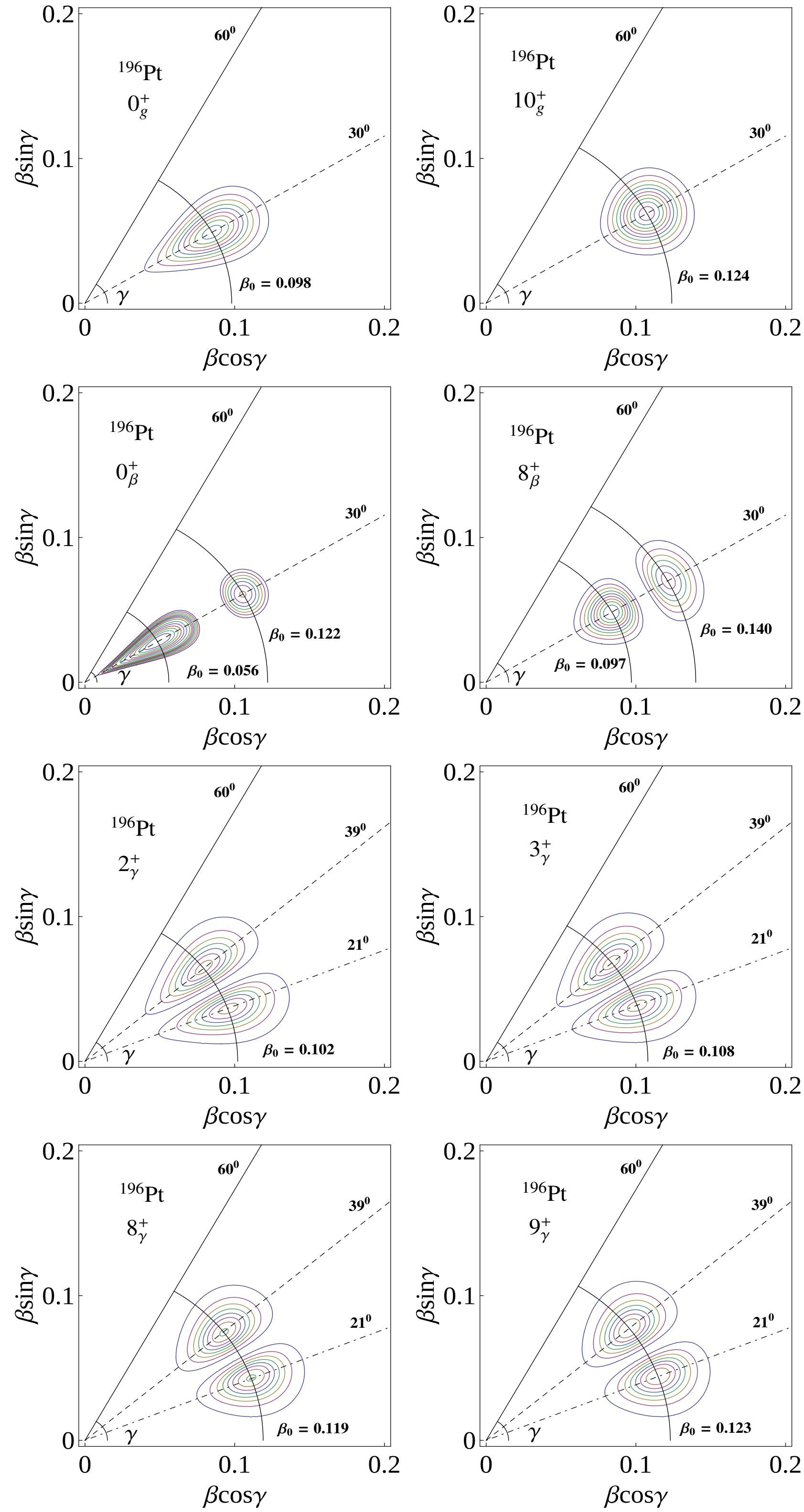

FIG. 7: The same as in Fig. 2 but for ^{196}Pt.

IV. CONCLUSIONS

In previous sections we studied the change of the nuclear shape along the isotope chan of Pt. The nuclei with a prolate minimum for γ are treated with the SSA formalism, while for those having a behavior closer to the triaxial picture, the approach SMA is used. The results for energies and B(E2) values are given in tables. Comparison with experimental data as well as with other models points out a better agreement with the corresponding experimental data for the present formalism. There are isotopes where two minima in γ coexists which results in exhibiting a shape coexistence.

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