Description of the $2\nu\nu\beta\beta$ decay within a fully renormalized pnQRPA approach with a restored gauge symmetry

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A many body Hamiltonian involving the mean field for a projected spherical single particle basis, the pairing interactions for alike nucleons and the dipole-dipole proton-neutron interactions in the particle-hole (ph) channel and the ph dipole pairing potential, is treated by the projected gauge fully renormalized proton-neutron quasiparticle random phase approximation (PGFRpnQRPA) approach. The resulting wave functions and energies for the mother and the daughter nuclei are used to calculate the $2\nu\beta\beta$ decay rate and the process half life. For illustration, the formalism is applied for the decay ^{100}Mo→ ^{100}Ru. The results are in good agreement with the corresponding experimental data. The Ikeda sum rule (ISR) is obeyed.

Double beta decay is one of the most exciting topic of nuclear physics since the rate of the process is obtained by combining formalisms of electroweak interaction with those yielding nuclear matrix elements. Due to this feature it represents a sensitive test for both collaborating fields. The $2\nu\beta\beta$ process is interesting by its own but is also very attractive because it constitutes a test for the nuclear matrix elements (m.e.) which are used for the process of $0\nu\beta\beta$ decay. Discovery of this process may provide an answer to the fundamental question, whether neutrino is a Mayorana or a Dirac particle. The subject development is reflected by several review papers [1–4]. This letter deals with the $2\nu\beta\beta$ process. The formalism yielding closest results to the experimental data is the proton-neutron random phase approximation ($pnQRPA$), which includes the particle-hole (ph) and particle-particle (pp)[5] as independent two body interactions. The second leg of the $2\nu\beta\beta$ process is very sensitive to changing the relative strength of the later interaction, denoted hereafter by g_{pp}. It is worth mentioning that the two body interaction of ph type is repulsive while that of pp nature is attractive. For a critical value of g_{pp}, the first root of the pnQRPA equation vanishes. This is a signal that the pnQRPA approach is no longer valid. Moreover, the g_{pp} value which corresponds to a transition amplitude which agrees with the corresponding experimental data is close to the mentioned critical value. That means that the result is not stable to adding corrections to the RPA picture. The first improvement for the pnQRPA was achieved in Refs.[6], by using a boson expansion (BE) procedure. Later on another procedure showed up, which renormalized the dipole two quasiparticle operators [7]. Such a renormalization is inconsistently achieved since the scattering operators are not renormalized. This lack of consistency was removed in Ref. [8, 9] where a fully renormalized pnQRPA is proposed.

Unfortunately, all higher pnQRPA procedures mentioned above have a common drawback of violating the Ikeda sum rule (ISR) by an amount of about 20-30% [10]. It is believed that such a violation is caused by the gauge symmetry breaking. A method for restoring this symmetry was formulated in Ref. [11].

Here the results of Ref.[11] are improved in three respects: a) aiming at providing a unitary description of the process for spherical and deformed emitters, we use the projected spherical single particle basis defined in Ref.[12–14]. b) the space of proton-neutron dipole configurations is split in three subspaces, one being associated to the single β^-, one to the β^+ process, and one spanned by the unphysical states. c) the correlations for the second leg of the process are mainly determined by the ph dipole-pairing term. The numerical application is made for the $2\nu\beta\beta$ process $^{100}Mo \rightarrow ^{100}Ru$. Aiming at a selfcontent presentation, we give few details.

According to Ref.[12] the projected spherical basis is:

$$\Phi^{IM}_{nlj}(d) = \mathcal{N}^I_{nlj} P^I_{MI}[|nljI\rangle\Psi_g] \equiv \mathcal{N}^I_{nlj}\Psi^{IM}_{nlj}(d), \qquad (1)$$

where P^I_{MK} denotes the angular momentum projection operator, $|nljm\rangle$ is the spherical shell model state and Ψ_g is an axially deformed coherent state describing the ground state of a phenomenological core in terms of quadrupole bosons $b^\dagger_{2\mu}, b_{2\mu}$:

$$\Psi_g = exp[d(b^+_{20} - b_{20})]|0\rangle_b. \qquad (2)$$

Here $|0\rangle_b$ denotes the vacuum state for the quadrupole bosons. The single particle energies ϵ^I_{nlj} are obtained by averaging a particle-core Hamiltonian with the corresponding basis states. In order to keep close to the Nilsson model, where on each Ω state one can distribute 2 nucleons, here we change the norm of the projected states such that this restriction holds:

$$\langle\Phi^{IM}_\alpha|\Phi^{IM}_\alpha\rangle = 1 \Longrightarrow \sum_M \langle\Phi^{IM}_\alpha|\Phi^{IM}_\alpha\rangle = 2. \qquad (3)$$

Thus, the wave functions used to calculate the m.e. should be multiplied with $\sqrt{2/(2I+1)}$.

We suppose that the states describing the nuclei involved in a $2\nu\beta\beta$ process are described by a many body Hamiltonian which may be written in the projected spherical basis as:

$$H = \sum_{\tau,\alpha,I,M} \frac{2}{2I+1}(\epsilon_{\tau\alpha I} - \lambda_{\tau\alpha})c^{\dagger}_{\tau\alpha IM}c_{\tau\alpha IM} - \sum_{\tau,\alpha,I,I'} \frac{G_\tau}{4}P^{\dagger}_{\tau\alpha I}P_{\tau\alpha I'} + 2\chi \sum_{\substack{pn;p'\\ n';\mu}} \beta^{-}_{\mu}(pn)\beta^{+}_{-\mu}(p'n')(-)^{\mu}$$
$$-\chi_1 \sum_{\substack{pn;p'\\ n';\mu}} \left(\beta^{-}_{\mu}(pn)\beta^{-}_{\mu}(p'n') + \beta^{+}_{-\mu}(p'n')\beta^{+}_{-\mu}(pn)\right)(-1)^{1-\mu}, \quad (4)$$

where $c^{\dagger}_{\tau\alpha IM}(c_{\tau\alpha IM})$ denotes the creation (annihilation) operator of one nucleon of the type $\tau(= p,n)$ in the state Φ^{IM}_{α}, with α being an abbreviation for the set of quantum numbers nlj. The Hamiltonian H contains the mean field term, the pairing interaction for alike nucleons and the Gamow-Teller dipole-dipole interaction in the ph channel and the ph dipole-pairing, characterized by the strengths χ and χ_1, respectively. Passing to the quasiparticle representation $(a^{\dagger}_{\tau IM}, a_{\tau IM})$ the first two terms of H are replaced by the independent quasiparticles term, $\sum E_{\tau I}a^{\dagger}_{\tau IM}a_{\tau IM}$, while the ph dipole-dipole and ph dipole-pairing interactions are expressed in terms of the dipole two qp and the qp density operators:

$$\begin{aligned} A^{\dagger}_{1\mu}(pn) &= \sum C^{I_p\ I_n\ 1}_{m_p\ m_n\ \mu}a^{\dagger}_{pI_pm_p}a^{\dagger}_{nI_nm_n}, \\ B^{\dagger}_{1\mu}(pn) &= \sum C^{I_p\ I_n\ 1}_{m_p\ -m_n\ \mu}a^{\dagger}_{pj_pm_p}a_{nI_nm_n}(-)^{I_n-m_n}, \end{aligned} \quad (5)$$

and their their Hermitian conjugates. In Ref.[8], we showed that all these operators can be renormalized as suggested by the commutation equations:

$$\begin{aligned} \left[A_{1\mu}(k), A^{\dagger}_{1\mu'}(k')\right] &\approx \delta_{k,k'}\delta_{\mu,\mu'}\left[1 - \frac{\hat{N}_n}{\hat{I}^2_n} - \frac{\hat{N}_p}{\hat{I}^2_p}\right], \\ \left[B^{\dagger}_{1\mu}(k), A^{\dagger}_{1\mu'}(k')\right] &\approx \left[B^{\dagger}_{1\mu}(k), A_{1\mu'}(k')\right] \approx 0, \\ \left[B_{1\mu}(k), B^{\dagger}_{1\mu'}(k')\right] &\approx \delta_{k,k'}\delta_{\mu,\mu'}\left[\frac{\hat{N}_n}{\hat{I}^2_n} - \frac{\hat{N}_p}{\hat{I}^2_p}\right], \ k = (I_p, I_n). \end{aligned} \quad (6)$$

Indeed, denoting by $C^{(1)}_{I_p,I_n}$ and $C^{(2)}_{I_p,I_n}$ the averages of the right hand sides of (7) with the renormalized RPA vacuum state, the renormalized operators defined as $\bar{A}_{1\mu}(k) = \frac{1}{\sqrt{C^{(1)}_k}}A_{1\mu},\ \bar{B}_{1\mu}(k) = \frac{1}{\sqrt{|C^{(2)}_k|}}B_{1\mu}$, obey boson like commutation relations:

$$\begin{aligned} \left[\bar{A}_{1\mu}(k), \bar{A}^{\dagger}_{1\mu'}(k')\right] &= \delta_{k,k'}\delta_{\mu,\mu'}, \\ \left[\bar{B}_{1\mu}(k), \bar{B}^{\dagger}_{1\mu'}(k')\right] &= \delta_{k,k'}\delta_{\mu,\mu'}f_k, \quad f_k = sign(C^{(2)}_k). \end{aligned} \quad (7)$$

Further, these operators are used to define the phonon operator:

$$C^{\dagger}_{1\mu} = \sum_k \left[X(k)\bar{A}^{\dagger}_{1\mu}(k) + Z(k)\bar{D}^{\dagger}_{1\mu}(k) - Y(k)\bar{A}_{1-\mu}(k)(-)^{1-\mu} - W(k)\bar{D}_{1-\mu}(k)(-)^{1-\mu}\right], \quad (8)$$

where $\bar{D}^{\dagger}_{1\mu}(k)$ is equal to $\bar{B}^{\dagger}_{1\mu'}(k')$ or $\bar{B}_{1\mu}(k)$ depending on whether f_k is + or -. The phonon amplitudes are determined by the equations supplied by the operator equations:

$$\left[H, C^{\dagger}_{1\mu}\right] = \omega C^{\dagger}_{1\mu} \quad \left[C_{1\mu}, C^{\dagger}_{1\mu'}\right] = \delta_{\mu\mu'}. \quad (9)$$

Interesting properties for these equations and their solutions are discussed in our previous publications [8, 9]. Here we mention one of these features. The renormalized ground state is a superposition of components describing the neighboring nuclei $(N-1, Z+1), (N+1, Z-1), (N+1, Z+1), (N-1, Z-1)$. The first two components conserve the total number of nucleons (N+Z) but violates the third component of isospin, T_3. By contrast, the last two components violates the total number of nucleons but preserve T_3. Actually, the last two components contribute to the violation of the ISR. One can construct linear combinations of the basic operators $A^{\dagger}, A, B^{\dagger}, B$ which excite the nucleus (N, Z) to the nuclei $(N-1, Z+1), (N+1, Z-1), (N+1, Z+1), (N-1, Z-1)$, respectively. These operators are actually the images of:

$$\begin{aligned} \mathcal{A}^{\dagger}_{1\mu}(pn) &= -\left[c^{\dagger}_p c_{\widetilde{n}}\right]_{1\mu}, \ \mathcal{A}_{1\mu}(pn) = -\left[c^{\dagger}_p c_{\widetilde{n}}\right]^{\dagger}_{1\mu}, \\ \mathbf{A}^{\dagger}_{1\mu}(pn) &= \left[c^{\dagger}_p c^{\dagger}_n\right]_{1\mu}, \ \mathbf{A}_{1\mu}(pn) = \left[c^{\dagger}_p c^{\dagger}_n\right]^{\dagger}_{1\mu}. \end{aligned}$$

trough the BV transformation. The later operators are involved in the proton-neutron pp interaction. At the gauge projected RPA level these terms do not contribute at all and therefore they have been ignored in the present work. In terms of the new operators, H becomes:

$$\begin{aligned} H &= \sum_{\tau jm} E_{\tau j}a^{\dagger}_{\tau jm}a_{\tau jm} \\ &+ 2\chi \sum_{pn,p'n';\mu} \sigma_{pn;p'n'}\mathcal{A}^{\dagger}_{1\mu}(pn)\mathcal{A}_{1\mu}(p'n') - \chi_1 \sum_{\substack{pn;p'\\ n';\mu}} \sigma_{pn;p'n'} \\ &\times \left(\mathcal{A}^{\dagger}_{1\mu}(pn)\mathcal{A}^{\dagger}_{1,-\mu}(p'n') + \mathcal{A}_{1,-\mu}(p'n')\mathcal{A}_{1\mu}(pn)\right)(-)^{1-\mu}, \\ \sigma_{pn;p'n'} &= \frac{2}{3\hat{I}_n\hat{I}_{n'}}\langle I_p||\sigma||I_n\rangle\langle I_{p'}||\sigma||I_{n'}\rangle. \end{aligned} \quad (10)$$

Here $E_{\tau I}$ denotes the quasiparticle energy. The equations of motion of the operators defining the phonon operator are determined by the commutation relations:

$$\left[\mathcal{A}_{1\mu}(pn), \mathcal{A}^{\dagger}_{1\mu'}(p'n')\right] \approx \delta_{\mu,\mu'}\delta_{j_p,j_{p'}}\delta_{j_n,j_{n'}} \times \left[U^2_p - U^2_n + \frac{U^2_n - V^2_n}{\hat{j}^2_n}\hat{N}_n - \frac{U^2_p - V^2_p}{\hat{j}^2_p}\hat{N}_p\right]. \quad (11)$$

The average of the r.h. side of this equation with the $PGFRpnQRPA$ vacuum state is denoted by:

$$\begin{aligned} D_1(pn) &= U^2_p - U^2_n + \frac{1}{2I_n+1}(U^2_n - V^2_n)\langle\hat{N}_n\rangle \\ &- \frac{1}{2I_p+1}(U^2_p - V^2_p)\langle\hat{N}_p\rangle. \end{aligned} \quad (12)$$

The equations of motion show that the two qp energies are renormalized too:

$$E^{ren}(pn) = E_p(U_p^2 - V_p^2) + E_n(V_n^2 - U_n^2). \qquad (13)$$

The space of pn dipole states, $\mathcal{S}$, is written as a sum of three subspaces defined as:

$$\begin{aligned} \mathcal{S}_+ &= \{(p,n)|D_1(pn) > 0, \;\; E^{ren}(pn) > 0, \}\,, \\ \mathcal{S}_- &= \{(p,n)|D_1(pn) < 0, \;\; E^{ren}(pn) < 0, \}\,, \\ \mathcal{S}_{sp} &= \mathcal{S} - (\mathcal{S}_+ + \mathcal{S}_-)\,. \end{aligned} \qquad (14)$$

In $\mathcal{S}_+$ one defines the renormalized operators:

$$\bar{\mathcal{A}}^\dagger_{1\mu}(pn) = \frac{1}{\sqrt{D_1(pn)}}\mathcal{A}^\dagger_{1\mu}(pn), \;\; \bar{\mathcal{A}}_{1\mu}(pn) = \left(\bar{\mathcal{A}}^\dagger_{1\mu}(pn)\right)^\dagger, \qquad (15)$$

while in $\mathcal{S}_-$ the renormalized operators are:

$$\bar{\mathcal{F}}^\dagger_{1\mu}(pn) = \frac{1}{\sqrt{|D_1(pn)|}}\mathcal{A}_{1\mu}(pn), \;\; \bar{\mathcal{F}}_{1\mu}(pn) = \left(\bar{\mathcal{F}}^\dagger_{1\mu}(pn)\right)^\dagger. \qquad (16)$$

Indeed, the operator pairs $\mathcal{A}_{1\mu}, \mathcal{A}^\dagger_{1\mu}$ and $\mathcal{F}_{1\mu}, \mathcal{F}^\dagger_{1\mu}$ satisfy commutation relations of boson type. An RPA treatment within $\mathcal{S}_{sp}$ would yield either vanishing or negative energies. The corresponding states are therefore spurious.

The operator equations:

$$\left[H, \Gamma^\dagger_{1\mu}\right] = \omega\Gamma^\dagger_{1\mu}, \;\; \left[\Gamma_{1\mu}, \Gamma^\dagger_{1\mu'}\right] = \delta_{\mu,\mu'}. \qquad (17)$$

define the new pnQRPA equation for the phonon amplitudes:

$$\begin{aligned} \Gamma^\dagger_{1\mu} &= \sum_k \Big[X(k)\bar{\mathcal{A}}^\dagger_{1\mu}(k) + Z(k)\bar{\mathcal{F}}^\dagger_{1\mu}(k) \\ &- \; Y(k)\bar{\mathcal{A}}_{1-\mu}(k)(-)^{1-\mu} - W(k)\bar{\mathcal{F}}_{1-\mu}(k)(-)^{1-\mu}\Big] \end{aligned} \qquad (18)$$

In order to solve the equations for the phonon amplitudes we need to know $D_1(pn)$ and, therefore, the averages of the qp's number operators, $\hat{N}_p$ and $\hat{N}_n$. These are written first in particle representation and then the particle number conserving term is expressed as a linear combination of $\mathcal{A}^\dagger\mathcal{A}$ and $\mathcal{F}^\dagger\mathcal{F}$ chosen such that their commutators with $\mathcal{A}^\dagger, \mathcal{A}$ and $\mathcal{F}^\dagger, \mathcal{F}$ are preserved. The final result is:

$$\begin{aligned} \langle\hat{N}_p\rangle &= V_p^2(2I_p+1) + 3(U_p^2 - V_p^2)\Big(\sum_{\substack{n',k \\ (p,n')\in\mathcal{S}_+}} D_1(p,n') \\ &\times (Y_k(p,n'))^2 - \sum_{\substack{n',k \\ (p,n')\in\mathcal{S}_-}} D_1(p,n')(W_k(p,n'))^2\Big), \\ \langle\hat{N}_n\rangle &= V_n^2(2I_n+1) + 3(U_n^2 - V_n^2)\Big(\sum_{\substack{p',k \\ (p',n)\in\mathcal{S}_+}} D_1(p',n) \\ &\times (Y_k(p',n))^2 - \sum_{\substack{p',k \\ (p',n)\in\mathcal{S}_-}} D_1(p',n)(W_k(p',n))^2\Big). \end{aligned} \qquad (19)$$

	k	G_p	G_n	ISR	$\log ft$	χ	χ_1
^{100}Mo	5.5	0.18	0.288	15.995	$^{100}\mathrm{Mo} \xleftarrow{\beta^+/EC} {}^{100}\mathrm{Tc}$	0.232	1.406
					$4.45^{+0.18}_{-0.30}$ $^{a)}$ 4.65		
^{100}Ru	5.5	0.15	0.255	12.002	$^{100}\mathrm{Tc} \xrightarrow{\beta^-} {}^{100}\mathrm{Ru}$	0.232	1.406
					4.66 $^{a)}$ 4.12		

TABLE I: The strengths for pairing interactions (G_p) and (G_n) the GT dipole (χ) and the ph dipole-pairing interaction (χ_1), given in MeV, used in our work. We also give the scaling factor k involved in the boson expression of $\alpha_{2\mu}$ as well as the resulting $\log ft$ values characterizing the β^+/EC and β_- transitions of ^{100}Tc.

The pnQRPA equations and (19) are to be solved iteratively. Note that using the qp representation for the basic operators $\mathcal{A}^\dagger_{1\mu}, \mathcal{F}^\dagger_{1\mu}, \mathcal{A}_{1,-\mu}(-1)^{1-\mu}, \mathcal{F}_{1,-\mu}(-)^{1-\mu}$, one obtains that $\Gamma^\dagger_{1\mu}$ involves the scattering pn operators. Thus, the present description is, indeed, a $PGFRpnQRPA$ approach.

The formalism presented above was used to describe the $2\nu\beta\beta$ process. If the energy carried by leptons in the intermediate state is approximated by the sum of the rest energy of the emitted electron and half the Q-value of the double beta decay process, $\Delta E = \frac{1}{2}Q_{\beta\beta} + m_ec^2$, the reciprocal value of the $2\nu\beta\beta$ half life can be factorized as: $(T^{2\nu\beta\beta}_{1/2})^{-1} = F|M_{GT}(0^+_i \rightarrow 0^+_f)|^2$, where F is an integral on the phase space, independent of the nuclear structure, while M_{GT} stands for the Gamow-Teller transition amplitude and has the expression :

$$M_{GT} = \sqrt{3}\sum_{k,k'}\frac{{}_i\langle 0||\beta^+_i||1_k\rangle_{ii}\langle 1_k|1_{k'}\rangle_{ff}\langle 1_{k'}||\beta^+_f||0\rangle_f}{E_k + \Delta E + E_{1^+}}. \qquad (20)$$

In the above equation, the denominator consists of three terms: a) ΔE, which was already defined, b) the average value of the k-th $PGFRpnQRPA$ energy normalized to the particular value corresponding to k=1, and c) the experimental energy for the lowest 1^+ state. The indices carried by the β^+ operators indicate that they act in the space spanned by the $PGFRpnQRPA$ states associated to the initial (i) or final (f) nucleus. The overlap m.e. of the single phonon states in the initial and final nuclei respectively, are calculated within $PGFRpnQRPA$. In Eq.(20), the Rose's convention for the reduced m.e. is used. Note that if we restrict the pn space to $\mathcal{S}_+$, M_{GT} vanishes due to the second leg of the transition. Also, we remark that the operator $\bar{\mathcal{A}}^\dagger_{1\mu}$ plays the role of a β^- transition operator, while when $\bar{\mathcal{F}}^\dagger_{1\mu}$ is applied on the ground state of the daughter nucleus, it induces a β^+ transition. Therefore, the 2β decay cannot be described by considering the β^- transition alone. The $2\nu\beta\beta$ is allowed even if the χ_1 interaction is missing but $\mathcal{N}_- \neq 0$.

For illustration, we present the results for the transition ^{100}Mo$\rightarrow$^{100}Ru. For this case the energy corrections involved in Eq.(20) are: $\Delta E = 2.026 MeV$, $E_{1^+} = 0.0 MeV$. The parameters defining the single particle energies are those of the spherical shell model, the de-

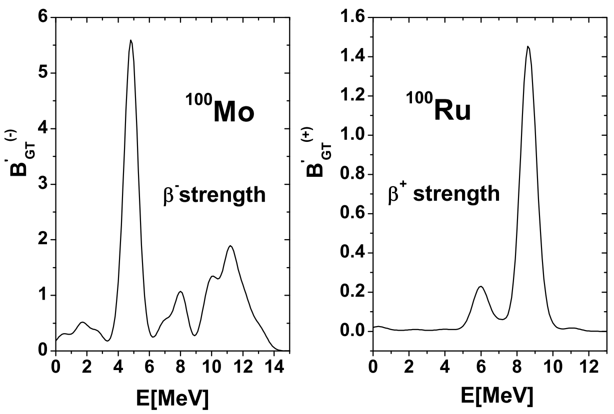

FIG. 1: One third of the single β^- strength for ^{100}Mo (left panel), and one third of the β^+ strength for ^{100}Ru, folded by a Gaussian with a width of 0.5MeV, are plotted as functions of the corresponding energies yielded by our approach. The difference $B_{GT}^{'(-)} - B_{GT}^{'(+)}$ is to be compared with the ISR value i.e.,$N-Z$.

formation parameter d and the parameter k relating the quadrupole coordinate with the quadrupole bosons. These are fixed as described in Ref.[14]. The core system is defined by $(Z,N)=(20,20)$. Labeling the states according to their energies ordering, the single particle space is defined by the indices interval $[11,55]$. The dimensions for the spaces $(\mathcal{S}_+,\mathcal{S}_-)$ are $(137,1)$ and $(139,2)$ for the mother and daughter nuclei, respectively. The dimension for $\mathcal{S}$ is 163 for the mother and 175 for the daughter nucleus. The strengths of the dipole pn and the ph dipole pairing interactions were taken so that the calculated $log\ ft$ values for the single β decay of ^{100}Tc are close to the corresponding experimental data. Using these input data we calculated the distribution of the $\beta^{\pm}$ strengths with the result shown in Fig.1. The energy intervals where both distributions are large, contribute significantly to the double beta transition amplitude.

Calculating first the GT transition amplitude and then the Fermi integral with $G_A=1.254$, as in Ref.[2], we obtained the result: $|M_{GT}|=0.221$, $T_{1/2}=8.79\cdot 10^{18}yr$. This result should be compared with the experimental results [15, 16]: $T_{1/2}=(8.0\pm 0.16)\cdot 10^{18}yr, T_{1/2}=(0.115^{+0.03}_{-0.02})\cdot 10^{20}yr$. Another experimental result concerns the summed strength for the β^- transition: $\sum B_{GT-}=26.69$. Quenching the theoretical result by a factor 0.6, as to account for the missing strength, one obtains the value of 28.96. The intermediate odd-odd nucleus, ^{100}Tc, can perform the transition β^+/EC, feeding ^{100}Mo, or the β^- transition to ^{100}Ru. The measured $\log ft$ values for these transitions, are given in Table 1. The theoretical results are obtained by:

$$ft_{\mp}=\frac{6160}{[{}_l\langle 1_1||\beta^{\pm}||0\rangle_l g_A]^2},\ l=i,f. \qquad (21)$$

In order to take account of the effect of distant states responsible for the "missing strength" in the giant GT resonance [2] we chose $g_A=1.0$.

Summarizing, one may say that restoring the gauge symmetry from the fully renormalized pnQRPA, one obtains a realistic description of the transition rate and moreover the ISR is obeyed. The attractive interaction of ph dipole-pairing type is responsible for the ground state correlations. To a less extent these are caused by the $\mathcal{F}$ components of the new phonon operator. The projection of gauge is essential for restoring the ISR. The gauge projection of the $pnQRPA$ was previously achieved in Ref.[17] where the ISR is anyway satisfied within $pnQRPA$. Therein the effect of projection is small.

Two GT resonances centered at 13.3 and 8. MeV respectively, and carrying a B(GT) strength of 23.1 ± 3.8 and 2.9 ± 0.5 were identified in Ref.[18]. Here the two centroids are at 11.25 and 8.0 MeV and the corresponding quenched strength are 11.16 and 4.23.Thus, the centroid energy and the corresponding strength of the main GTR, are only qualitatively explained. The data reflects a relative large χ_1 interaction, which transfers a large amount of strength to the resonance shown at 5 MeV.
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