Dedicated to Professor Oliviu Gherman's 80th Anniversary

A PHENOMENOLOGICAL INTERPRETATION OF 4^+ AND 6^+ STATE MULTIPLETS IN EVEN-EVEN NUCLEI

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The procedure used in a previous publication [11] to describe the multiplets 0^+ and 2^+ was extended to two other multiplets 4^+ and 6^+. Using the same parameters as in the quoted reference we calculated the semiclassical energies for the new multiplets and results were compared with the corresponding experimental data. Alternatively, the energies are considered as eigenvalues of the model Hamiltonian with the already fixed structure coefficients. This method works pretty well for ^{154}Gd but fails for the rest of nuclei. For these nuclei the fitting procedure for the parameters defining the model Hamiltonian must be applied by considering all energy levels from the four multiplets.

1. INTRODUCTION

The collective states of deformed nuclei are usually classified in rotational bands distinguished by a quantum number K, which is the angular momentum projection on the z axis of the intrinsic reference frame. The collective character of the states is diminished by increasing the value of K [1–4]. In Ref. [5] one of us (A.A.R.) suggested a possible method of developing bands in a *horizontal* fashion. Indeed, therein on the top of each state in the ground band a full band of monopole multi-phonon states has been constructed. This idea was recently considered in a phenomenological context by trying to organize the states describing the motion of the intrinsic degrees of freedom, in bands. Thus, two intrinsic collective coordinates, similar to the nuclear deformations β and γ, are described by the irreducible representations of a SU(2) group acting in a fictitious space (i.e. not in ordinary space) . Compact formulas for excitation energies have been obtained [6, 7].

Later on we addressed the question whether these expressions could provide a realistic parametrization of the data. We were interested to describe about 26 states 0^+ and 67 states 2^+ observed in ^{168}Er by means of the (p, t) reaction [8]. In the cited paper the excitation energies and the corresponding reaction strengths were provided. These data were described qualitatively by two microscopic models, called projected shell model (PSM) and quasiparticle phonon model (QPM), respectively.

Both models have some inherent drawbacks. PSM restricts the fermion space to four quasiparticle states and even from the four qp space the states with four alike quasiparticles are excluded. This is not the case of QPM where the multi-quasiparticle components are taken into account by means of the QRPA approach. However, the final states contain at most two phonon states. These states violate the Pauli principle and moreover are not states of good angular momentum.

The first attempt to fit the data of Ref. [8] was made in Ref. [9] by using a phenomenological model which was earlier developed in Ref. [7]. We used alternatively two boson Hamiltonians including high anharmonicities, one being treated semi-classically while the other one being diagonal in the standard quadrupole boson basis, $|NV\alpha JM\rangle$. The parameters involved in the model Hamiltonian were fixed by a least mean square procedure. A nice agreement between the calculated energies for the two sets of states 0^+ and 2^+ has been obtained. Although the experimental data for the E2 transitions linking the states 0^+ and 2^+ are lacking we presented closed formulas for them.

After the quoted paper showed up another 12 new states 2^+ have been identified in the same nucleus (^{168}Er) by a more careful examination of the data from the (p,t) experiment [10]. This was certainly a challenge for us to check whether the new data could be described with the earlier fitted parameters. Amazingly the new data fell on the theoretical curves for energies. This was a good reason to extend our boson description to some other nuclei. The results were all positive and described in Ref. [11].

Certainly some of the considered states have a pronounced two (or many) quasiparticle character. The question which we however raise is whether these can be mimicked by the phenomenological multiphonon states.

The aim of this paper is to investigate whether the compact energy formulas can be also used for higher angular momentum multiplets like 4^+ and 6^+. The simplest path to follow is to start with those nuclei considered in Ref. [11] and use the parameters determined in the previous analysis for the states 0^+ and 2^+. Then, in a subsequent work we shall make a systematic study of the states $0^+, 2^+, 4^+, 6^+$ and point out the global features (if any).

For the sake of completeness and, at a time, for facilitating the presentation of the results we describe very briefly the formalism which leads us to the compact formulas we are going to use.

Thus the plan for our exposure is as follows. In Section 2 we present the semi-classical formalism while the boson description given in Section 3. Numerical analysis is made in Section 4 while the final conclusions are drawn in Section 5.

2. SEMICLASSICAL DESCRIPTION

Although more details on the model can be found in Ref. [7], we give here the essential information to make the presentation self-contained. We use a sixth-order quadrupole boson Hamiltonian:

$$H = \epsilon\hat{N} + \sum_{J=0,2,4} C_J \left[\left(b_2^\dagger b_2^\dagger \right)_J (b_2 b_2)_J \right]_0 + F \left(b_2^\dagger b_2^\dagger \right)_0 \hat{N} \left(b_2 b_2 \right)_0 . \tag{1}$$

where $b_{2\mu}^\dagger, b_{2\mu}$, with $-2 \le \mu \le 2$, are the quadrupole boson operators and $\hat{N}$ the boson number operator. Averaging H on a coherent state for the bosons $b_{20}^\dagger$ and $\frac{1}{\sqrt{2}}(b_{22}^\dagger + b_{2,-2}^\dagger)$,

$$|\Psi\rangle = \exp\left[z_0 b_{20}^\dagger + z_2 (b_{22}^\dagger + b_{2,-2}^\dagger) - h.c) \right] |0\rangle, \tag{2}$$

one obtains a classical Hamilton function, $\mathcal{H}$, depending on two canonical coordinates, q_1 and q_2, and their corresponding conjugate momenta.

$$\begin{aligned} q_i &= 2^{(k+2)/4} Re(z_k),\ p_i = \hbar 2^{(k+2)/4} Im(z_k), \\ k &= 0, 2,\ i = \frac{k+2}{2}. \end{aligned} \tag{3}$$

The expression of $\mathcal{H}$ is:

$$\begin{aligned} \mathcal{H} = &\frac{A}{2}\left(q_1^2 + q_2^2 + \frac{1}{\hbar^2}(p_1^2 + p_2^2) \right) + \frac{B}{4}\left(q_1^2 + q_2^2 + \frac{1}{\hbar^2}(p_1^2 + p_2^2) \right)^2 + \\ &+ \frac{C}{8\hbar^2}(q_1 p_2 - q_2 p_1)^2 + \frac{F}{10}\left[\frac{1}{4}\left(q_1^2 + q_2^2 + \frac{1}{\hbar^2}(p_1^2 + p_2^2) \right)^2 - \frac{1}{\hbar^2}(q_1 p_2 - q_2 p_1)^2 \right] \\ &\times \left(q_1^2 + q_2^2 + \frac{1}{\hbar^2}(p_1^2 + p_2^2) \right). \end{aligned} \tag{4}$$

The new coefficients A, B, C, F depend on the parameters ϵ, C_J involved in the initial boson Hamiltonian.

$$A = \epsilon,\ B = \frac{1}{5}C_0 + \frac{2}{7\sqrt{5}}C_2 + \frac{6}{35}C_4,\ C = -\frac{8}{5}C_0 + \frac{16}{7\sqrt{5}}C_2 - \frac{8}{35}C_4. \tag{5}$$

The classical Hamilton function describes a system of two degrees of freedom or, in other words a particle moving in a plane. Basically it contains a kinetic and a potential term plus a coupling term depending on both the coordinates and the conjugate momenta. The potential energy depends only on coordinates and has a simple expression:

$$V(q_1, q_2) \equiv \mathcal{H}|_{p_1 = p_1 = 0} = \frac{A}{2}(q_1^2 + q_2^2) + \frac{B}{4}(q_1^2 + q_2^2)^2 + \frac{F}{40}(q_1^2 + q_2^2)^3. \tag{6}$$

$\mathcal{H}$ contains two distinct terms describing an anharmonic motion of a classical plane oscillator and a pseudo-rotation around an axis perpendicular to the oscillator plane, respectively. Taking into account that the third component of the pseudo-angular momentum is a constant of motion, the classical Hamiltonian in the reduced space can be easily quantized and the resulting energy is:

$$\epsilon_{n,M} = A(n+1) + B(n+1)^2 + \frac{C}{2}M^2 + \frac{F}{5}\left[(n+1)^3 - 4(n+1)M^2\right], \qquad (7)$$

The number of the plane oscillator quanta is denoted by n while the value of the third component of the pseudo-angular momentum is M. Actually, Eq. (7) represents a semi-classical spectrum which describes the motion of the intrinsic degrees of freedom which are related to those introduced by the liquid drop model, i.e. β and γ. Assuming that the rotational degrees of freedom are only weakly coupled to the motion of the intrinsic coordinates, the total energy associated to the motion in the laboratory frame can be written as a sum of two terms corresponding to the intrinsic and rotational motion, respectively:

$$\epsilon_{n,M,J} = A(n+1) + B(n+1)^2 + \frac{C}{2}M^2 + \frac{F}{5}\left[(n+1)^3 - 4(n+1)M^2\right] + \delta J(J+1) \qquad (8)$$

This expression for energies is a generalization of that obtained in Refs. [12, 13] where the energies from the ground state band were represented as a weighted sum of a vibrational and a rotational term. In Ref. [7] it was shown that the components of the pseudo-angular momentum $\vec{L}$, are obtained by averaging the generators of a boson $SU(2)$ algebra with the coherent state given by Eq. (2). An explicit expression of the angular momentum in the laboratory frame and the pseudo-angular momentum expressed in terms of the intrinsic coordinates is given in Ref. [7]. Considering that the pseudo-angular momentum is perpendicular to the (q_1, q_2) plane, i.e., M=L, the above mentioned relation between L and J suggests the values $M = 2$ when $J = 4$ and $M = 3$ for $J = 6$. Alternatively, we could chose $M = \sqrt{L(L+1)}$ which leads to $M = \sqrt{6}$ and $\sqrt{12}$ for J=4 and J=6 respectively. The results presented here correspond to the first option.

With these restrictions the excitation energies for the states 4^+ and 6^+ become:

$$\begin{aligned}
E_{n,4} \equiv \epsilon_{n,2,4} - \epsilon_{0,0,0} =& \frac{1}{5}Fn^3 + (\frac{3}{5}F + B)n^2 + (A + 2B - \frac{13}{5}F)n \\
&+2C - \frac{16}{5}F + 20\delta,\ n \geq 2, \\
E_{n,6} \equiv \epsilon_{n,3,6} - \epsilon_{0,0,0} =& \frac{1}{5}Fn^3 + (\frac{3}{5}F + B)n^2 + (A + 2B - \frac{33}{5}F)n \\
&+\frac{9}{2}C - \frac{36}{5}F + 42\delta,\ n \geq 3.
\end{aligned} \qquad (9)$$

3. BOSON DESCRIPTION

We note that the Hamiltonian introduced in the previous section is a boson number conserving Hamiltonian involving anharmonicities up to the sixth order. Despite this feature we showed that one could obtain analytical expressions for its eigenvalues. In order to prove that we write the Hamiltonian in a slightly different form using the results of Ref.[7] concerning the fourth order boson term:

$$\begin{aligned} H =&(A+\gamma)\hat{N}+(B+\frac{C}{8})\hat{N}^2-\frac{1}{6}\left(B+\frac{C}{8}+\gamma\right)\hat{J}^2 \\ &-\frac{5}{8}C\left(b_2^\dagger b_2^\dagger\right)_0(b_2b_2)_0+F\left(b_2^\dagger b_2^\dagger\right)_0\hat{N}\,(b_2b_2)_0. \end{aligned} \tag{10}$$

where the coefficient γ has the expression:

$$\gamma=\frac{2}{7\sqrt{5}}C_2-\frac{3}{7}C_4. \tag{11}$$

This Hamiltonian commutes with the operators $\hat{N},\hat{\Lambda},\hat{J}^2,\hat{J}_z$ where $\hat{\Lambda}$ denotes the Casimir operator of the group R_5:

$$\hat{\Lambda}=\hat{N}(\hat{N}+3)-5(b_2^\dagger b_2^\dagger)_0(bb)_0. \tag{12}$$

The Hamiltonian is diagonal in the basis $|Nv\alpha JM\rangle$, where the specified quantum numbers are the number of bosons, seniority, missing quantum number, angular momentum and its projection on the z axis, respectively.

We just mention that this basis was explicitly constructed in both the laboratory and intrinsic frames in Ref. [14]. The analytical form of this basis seems to be very useful to calculate the matrix elements of any operator which is monomial in quadrupole bosons [15]. The quantum number α is called the missing quantum number due to the degeneracy appearing in the reduction from the group $R(5)$ to the group $R(3)$. The number of degenerate states was analytically calculated in Ref. [16].

The eigenvalues of H in the mentioned basis are:

$$\begin{aligned} E_{N,v,J}=&\frac{1}{5}FN^3+(B+\frac{1}{5}F)N^2+(A+\gamma-3(\frac{1}{8}C+\frac{2}{5}F))N \\ -&\frac{1}{6}(B+\frac{1}{8}C+\gamma)J(J+1)+(\frac{1}{8}C+\frac{2}{5}F)v^2+3(\frac{1}{8}C+\frac{2}{5}F)v-\frac{1}{5}FNv^2-\frac{3}{5}FNv. \end{aligned}$$

By contrast to the semi-classical energies given by Eq.(8), the eigenvalues of H are characterized by two quantum numbers, N and v. Due to this feature one expects that the new expression for energies yields a better description of the data.

We attempt to describe the available data for 4^+ and 6^+ by the lowest two seniority states. Since $N-v$ must be even, for each angular momentum we distinguish states described by even and by odd N, respectively. Thus, for J=4 and J=6 the above equation becomes:

$$E_{N,2,4} = \frac{1}{5}FN^3 + \left(B+\frac{1}{5}F\right)N^2 + \left(A+\gamma-\frac{3}{8}C-\frac{16}{5}F\right)N - \frac{5}{3}\left(2B-\frac{1}{2}C+2\gamma-\frac{12}{5}F\right),$$
$$E_{N,3,4} = \frac{1}{5}FN^3 + \left(B+\frac{1}{5}F\right)N^2 + \left(A+\gamma-\frac{3}{8}C-\frac{24}{5}F\right)N - \frac{5}{3}\left(2B-\frac{11}{10}C+2\gamma\right) + \frac{36}{5}F,$$
$$E_{N,3,6} = \frac{1}{5}FN^3 + \left(B+\frac{1}{5}F\right)N^2 + \left(A+\gamma-\frac{3}{8}C-\frac{24}{5}F\right)N + \left(-7B+\frac{11}{8}C-7\gamma+\frac{36}{5}F\right),$$
$$E_{N,4,6} = \frac{1}{5}FN^3 + \left(B+\frac{1}{5}F\right)N^2 + \left(A+\gamma-\frac{3}{8}C-\frac{34}{5}F\right)N - 7\left(B-\frac{3}{8}C+\gamma-\frac{8}{5}F\right) \tag{13}$$

It is worth mentioning that the sets of energies of the same seniority but different angular momentum have the same N dependence. Consequently the difference $E_{N,3,6} - E_{N,3,4}$ is independent of N. From Eqs. (13) one finds very simple expressions for the relative energies characterizing the multiplets 4^+ and 6^+:

$$\begin{aligned} E_{N,2,4} - E_{N,3,4} &= \frac{8}{5}F(N-2) - C, \\ E_{N,3,6} - E_{N,4,6} &= 2F(N-2) - \frac{5}{4}C, \\ E_{N,3,6} - E_{N,3,4} &= -\frac{11}{3}\left(B+\gamma+\frac{1}{8}C\right), \\ 5(E_{N,2,4} - E_{N,3,4}) &= 4(E_{N,3,6} - E_{N,4,6}). \end{aligned} \tag{14}$$

These simple equations might be useful when the parameters involved in the model Hamiltonian, i.e. B, C, γ, F, are to be determined.

4. NUMERICAL RESULTS

As we mentioned before, the main aim of this paper is to extend our study of the low spin multiplets achieved in Ref. [11] to those of angular momenta 4 and 6. The simplest way for our extension is to keep the same parameters as were determined in Ref. [11] and use the defining equations from the previous section. In the case of the semiclassical description the fitting procedure applied to the multiplets $0^+$and 2^+ has determined the parameters A, B, F and

$$\mathcal{C} = \frac{1}{2}C - \frac{4}{5}F + 6\delta. \tag{15}$$

Therefore the parameters C and δ could not be determined at a time. From Eq.(15) we can express C in terms of δ and thus we are left with one free parameter which is determined by a least square procedure of the data concerning the multiplets 4^+ and 6^+. The values obtained in this way for δ are given in Table I. Note that δ yielded

Table 1.

The parameter δ yielded by the fitting procedure for the nuclei under consideration.

	^{152}Gd	^{154}Gd	^{168}Er	^{180}W	^{184}W	^{190}Os
-δ[keV]	98.914	22.218	170.406	413.275	324.852	369.870

by the fitting procedure is a negative quantity. One may think that this feature is forbidden given its significance of half the reciprocal value of the moment of inertia. Actually this is not true since in the rotational limit the average of the model Hamiltonian on the angular momentum projected state generated by the coherent state $|\Psi\rangle$ yields a rotational term which is to be added to the δ term.

The result of this simple exercise is that the semiclassical description works for all nuclei considered in the previous publication, i.e. ^{152}Gd, ^{154}Gd, ^{168}Er, ^{180}W, ^{184}W and ^{190}Os where the multiplets 0^+ and 2^+ where studied. As for the boson description it turns out that only for ^{154}Gd the set of parameters provided by the analysis of the multiplets 0^+ and 2^+, is suitable. For the remaining cases a general fit considering all multiplets $0^+, 2^+, 4^+, 6^+$ at a time is needed.

In general the states 4^+ and 6^+ in the nuclei mentioned above, are populated in various experiments like β^- and ϵ decays, inelastic scatterings of the types (d,d'), (n,n'), (p,p'), (γ,γ'), Coulomb excitations, transfer reactions such as (p,t), (d,p), (t,p) as well as $(\alpha, Xn\gamma)$, $(p, Xn\gamma)$ or neutron capture reaction, (n,γ).

In what follows we shall discuss each nucleus separately. The states 4^+ in ^{152}Gd having the energies shown in Fig.1 were populated by the β^- decay of ^{152}Er or by the inelastic scattering (d,d') [17, 18].

The results for ^{154}Gd are plotted in Fig. 2 for the states 4^+ and Fig.3 for 6^+. For this case both methods of semiclassical approach and exact eigenvalues are presented. The experimental data are those from Refs. [18, 19]. The states were excited by several experiments from which we mention the capture of thermal neutrons, ^{153}Gd (n,γ), the ϵ decay of ^{154}Tb and the reaction ^{152}Sm (α, 2nγ).

For ^{168}Er the energies of 17 states 4^+ and 7 states 6^+ are available [18, 20]. These states were observed in several experiments such as ^{166}Er (t,p), ^{167}Er(n,γ) (E=24 keV resonance), ^{167}Er(d,p), (t,d) for the 4^+ states while the 6^+ states were observed in the reactions ^{167}Er(n,γ) (thermal neutrons), ^{170}Er (p,t), ^{167}Er (p,t), (t,d). In Fig. 4 we observe a good agreement between the semiclassical energies and the corresponding experimental data.

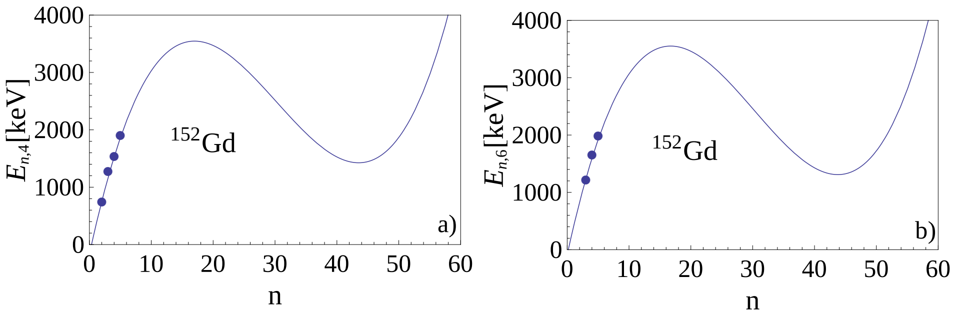

Fig. 1 – Excitation energies of the states $J^{\pi} = 4^{+}$ (a) and $J^{\pi} = 6^{+}$ (b) in ^{152}Gd. Theoretical results (full curve) are obtained using Eq. (9) and, for the involved parameters, the values obtained in the previous publication [11] by a fitting procedure applied to the multiplets 0^{+} and 2^{+}. The filled circles are experimental data points from Refs. [17, 18].

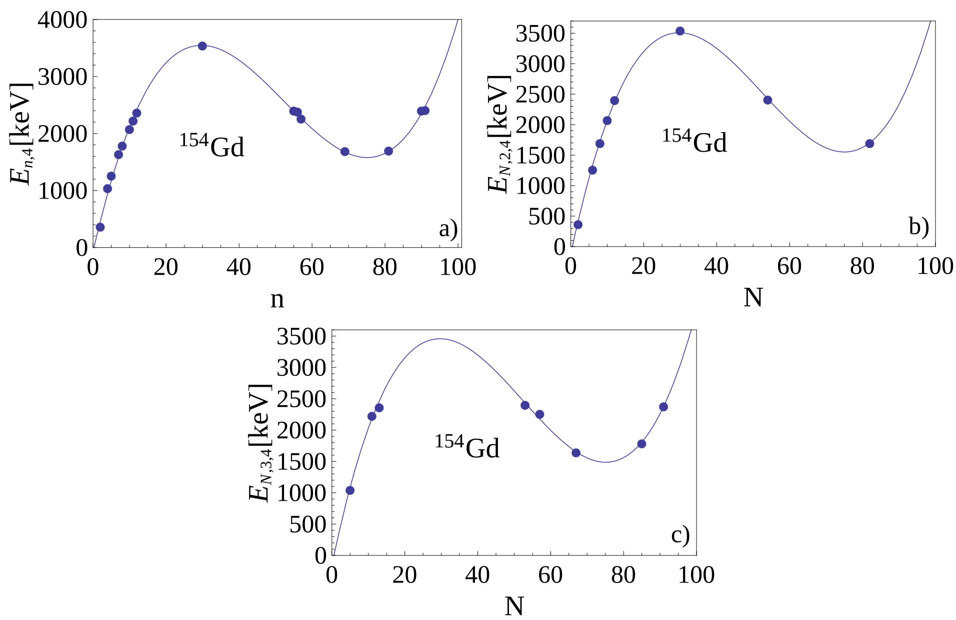

Fig. 2 – Semiclassical energies for the multiplet 4^{+} (full line) are compared in panel (a) with experimental data (discs) for ^{154}Gd from Refs. [18, 19]. In panels (b) and (c) the theoretical results are obtained with the exact eigenvalues $E_{N,2,4}$ and $E_{N,3,4}$ respectively, given by Eqs. 13.

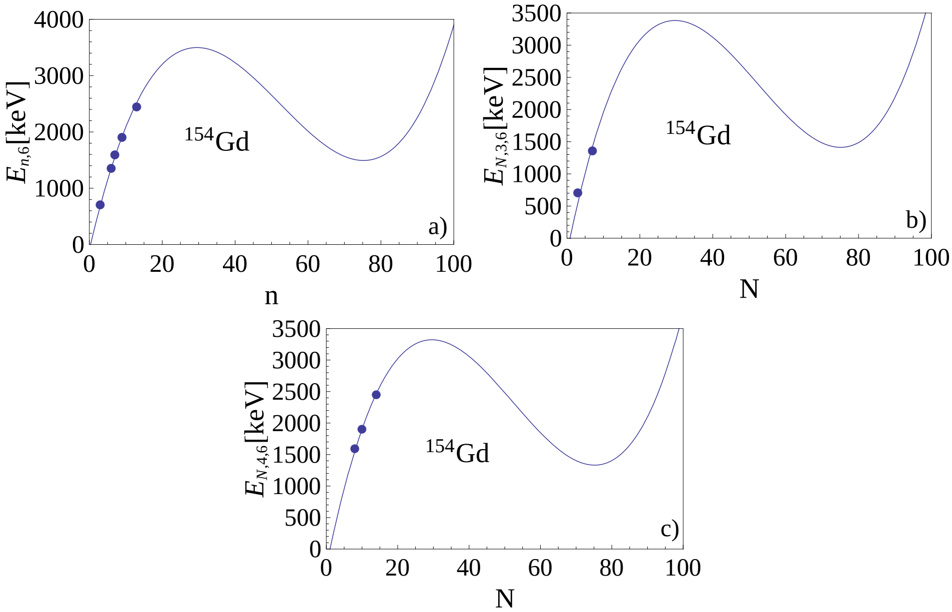

Fig. 3 – Semiclassical energies for the multiplet 6^+ (full line) are compared in panel a) with experimental data for ^{154}Gd (filled circles). In panels b) and c) the theoretical results are obtained with the exact eigenvalues denoted by $E_{N,3,6}$ and $E_{N,4,6}$ respectively, and given by Eqs. 13. Experimental data (filled circles) are from Refs. [18, 19].

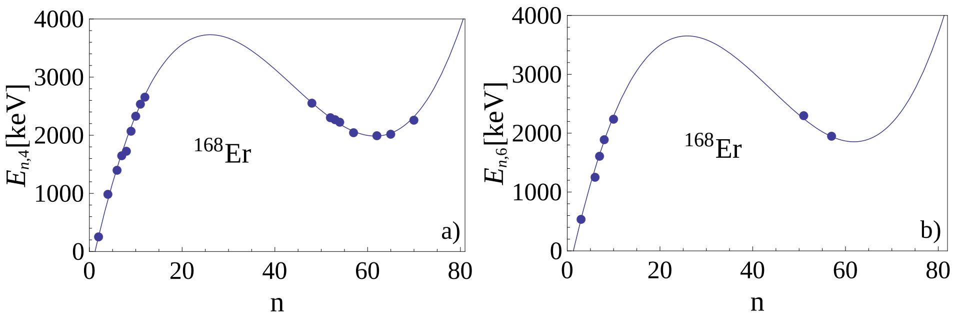

Fig. 4 – The same as in Fig. 1 but for ^{168}Er. Experimental data are from Refs. [18, 20].

For the two isotopes of W which are considered here only few energies are known [18, 21], namely a total of 5 states for ^{180}W and 6 for ^{184}W. They are compared with the semiclassical energies in Fig. 5. For ^{180}W the states 4^+ and 6^+ were populated by the experiment $^{181}Ta(p, 2n\gamma)$ (E=17 MeV). Concerning ^{184}W the states 4^+ and 6^+ were excited either by the β^- decay of ^{184}Ta or by the inelastic scattering ^{184}W(d,d').

In ^{190}Os four states 4^+ and three states 6^+ are known from experiments like β^- decay of ^{190}Re, thermal neutron capture ^{189}Os(n,γ), Coulomb excitation. The comparison between the semiclassical energies and the corresponding experimental data is performed in Fig. 6. It is instructive to see how the potential energy defined in the previous section, depends on the deformation dynamic variable r defined by:

$$r^2 = q_1^2 + q_2^2. \tag{16}$$

The deformation dependence of the potential energies associated to the nuclei under consideration is shown in Fig. 7.

The plot for potential energy looks similar to that for the n dependence of the excitation energy. However in the later case the quantum number n is obtained by quantizing the plane oscillator energy which includes both the potential and kinetic energy. Comparing the plots for excitation energy and that of potential energy, for each nucleus, one sees that in some cases there are states located in the secondary minimum. This allows us to state that the nuclei in such states belong to a distinct nuclear phase. Therefore these states exhibit signatures for a new nuclear phase. It is customary to consider the energy ratio E_{4^+}/E_{2^+} as a signature of the nuclear phase to which the considered nucleus belongs. In Fig. 8 a) we have represented the energy ratio for the states $|N, 2, 4\rangle$ and $|1, 1, 2\rangle$ in the boson description. In a harmonic picture one expects that the energy of a N boson state to be about N times the energy of the first state 2^+. Therefore the ratio should be equal to N in the harmonic limit. In the rotational limit for N=2 the ratio is 3.33. One notes that up to very large values of N the ratio is smaller than N which reflects a strong effect coming from anharmonicities. Another energy ratios plotted in Fig. 8 b) as a function of N concerns the states $|N, 2, 4\rangle$ and $|N-1, 1, 2\rangle$. In the harmonic limit this is equal to $N/(N-1)$ and therefore a larger than unity number. Large deviations from harmonic limit show up in the interval of N where the energy ratio is less than unity.

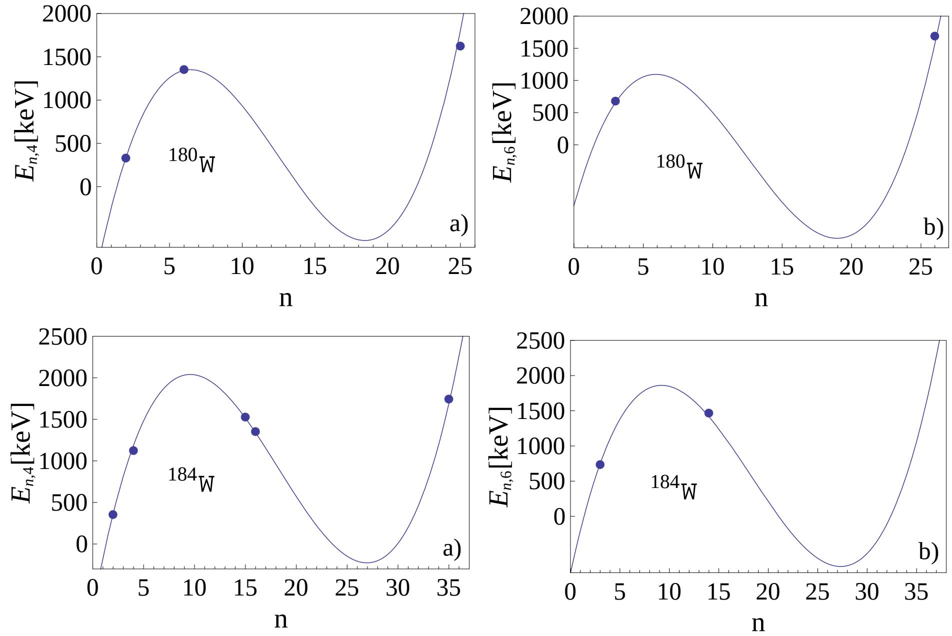

Fig. 5 – The same as in Fig.1 but for ^{180}W in the upper panels a) and b) and for ^{184}W in the lower panels a) and b). Experimental data for ^{180}W are from Refs. [18, 21] while for ^{184}W are from Refs. [18, 22].

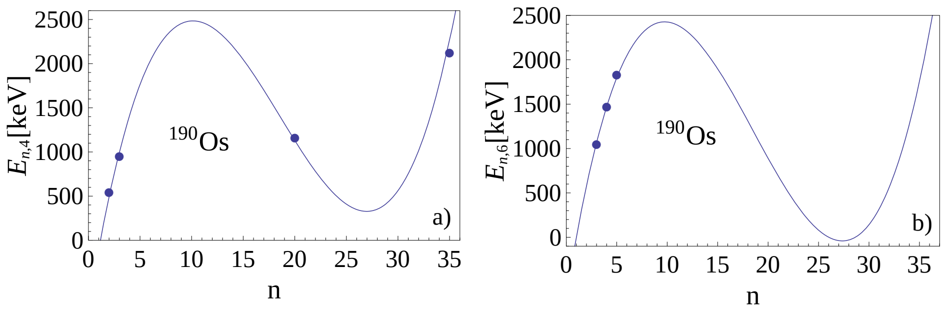

Fig. 6 – The same as in Fig. 1 but for ^{190}Os. Experimental data are from Refs. [18, 23].

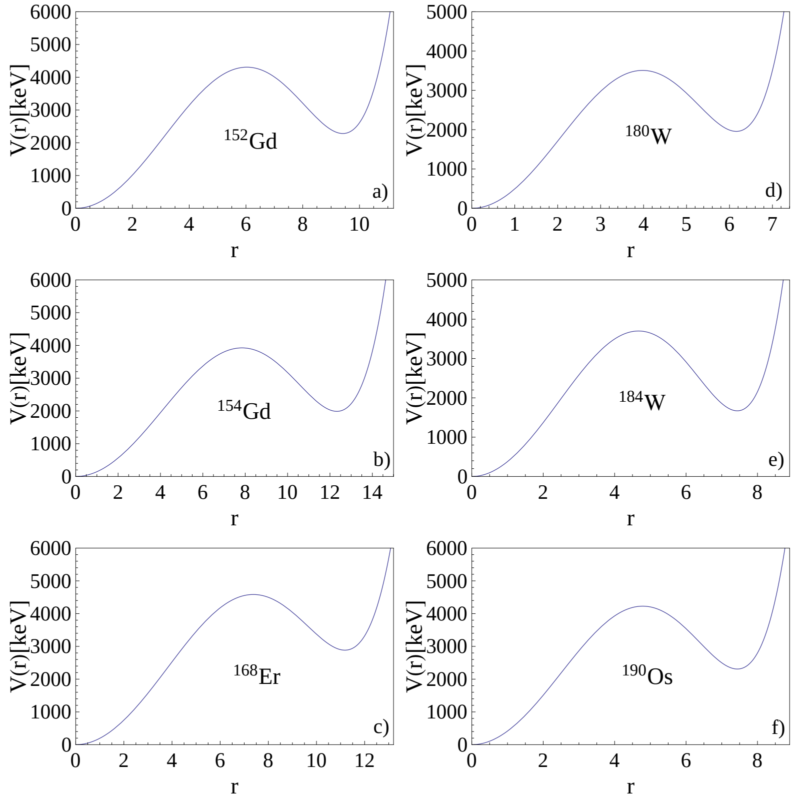

Fig. 7 – The potential energy term defined by Eq. (6) is plotted as function of the deformation dynamic variable r defined by Eq.(16) .

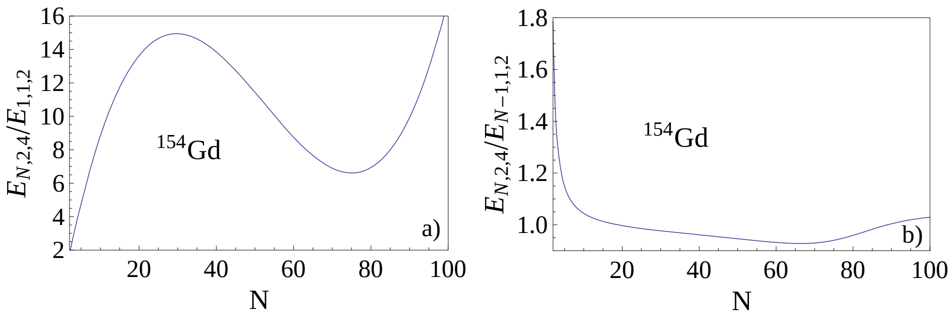

Fig. 8 – The ratios $E_{N,2,4}/E_{1,1,2}$ and $E_{N,2,4}/E_{N-1,1,2}$ are plotted as function of N, the number of bosons, for ^{154}Gd in panels a) and b), respectively.

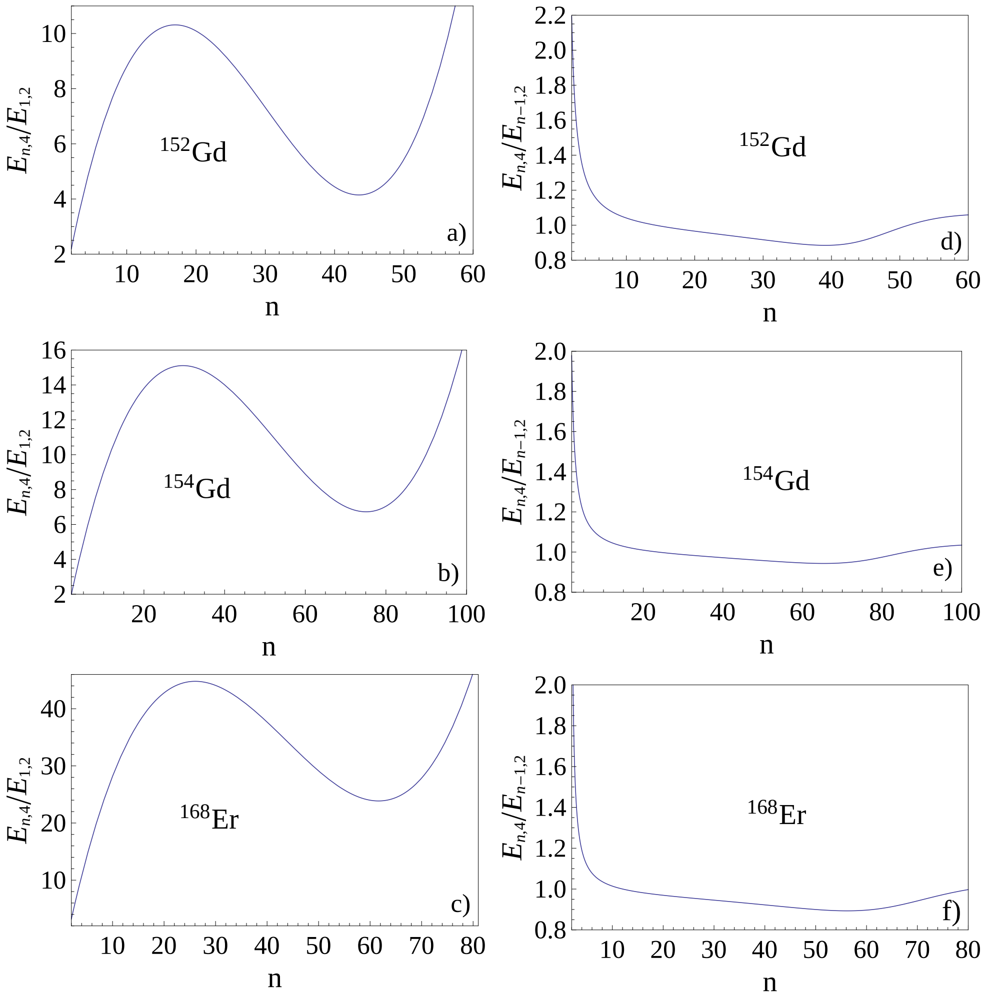

Fig. 9 – The ratios $E_{n,4}/E_{1,2}$ (left column) and $E_{n,4}/E_{n-1,2}$ (right column), are plotted as function of n for three nuclei: ^{152}Gd (a) and (d), ^{154}Gd (b) and (e) and ^{168}Er (c) and (f).

A similar study can be performed also within the semiclassical formalism. Indeed in Figs. 9 and 10 we plotted the ratios $E_{n,4}/E_{1,2}$ and $E_{n,4}/E_{n-1,2}$ as function of n, respectively.

Comparing the energy ratios given by the boson formalism with those given by the semiclassical approach for ^{154}Gd we see that they are similar. Also it is interesting to notice that the two ratios considered in the semiclassical picture exhibit minima for the same quantum numbers n. Except for ^{154}Gd where the minimum ratio $E_{n,4}/E_{n-1,2}$ is very flat, the minima for other nuclei are well pronounced.

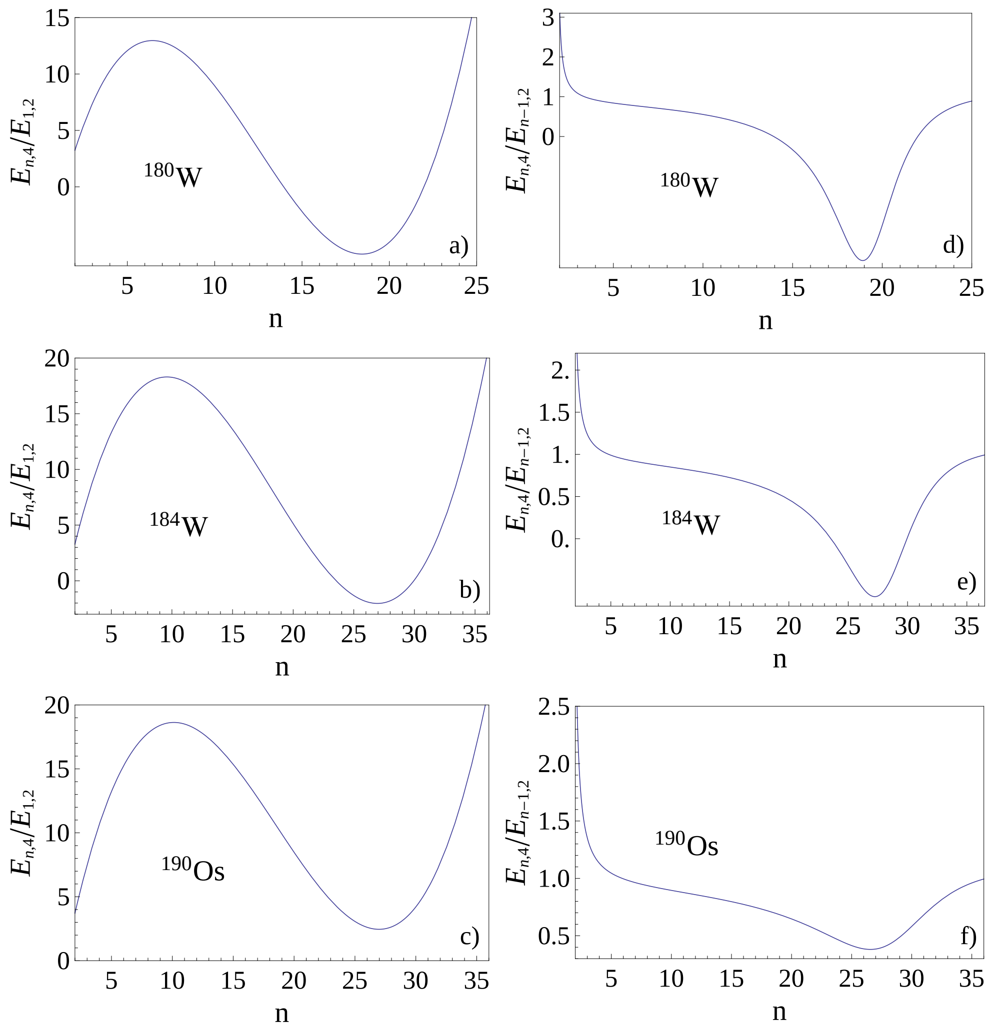

Fig. 10 – The ratios $E_{n,4}/E_{1,2}$ (left column) and $E_{n,4}/E_{n-1,2}$ (right column), are plotted as function of n for three nuclei: ^{180}W (a) and (d), ^{184}W (b) and (e) and ^{190}Os (c) and (f).

5. CONCLUSIONS

In the previous Sections we extended, with a positive result, the semiclassical description of the multiplets 0^+ and 2^+ to two other multiplets 4^+ and 6^+. Using the same parameters for the model Hamiltonian as in Ref. [11] the calculated energies for the multiplet members agree quite well with the corresponding experimental data. We tried to apply a similar extension for the alternative description where the energies are eigenvalues of the boson Hamiltonian. Our attempt was only partially successful. Indeed, good results, which are in agreement with the experimental data, are obtained only for ^{154}Gd. As for the remaining nuclei it is necessary to consider a simultaneous fitting procedure for all multiplets 0^+, 2^+, 4^+ and 6^+. We hope that our results will stimulate nuclear structure groups to study these multiplets. Obviously, the formalisms employed would be essentially different from those suitable for the description of the rotational bands.

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