HEAVY ION ORBITING AND REGGE POLES (I)

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We review the semiclassical theory for heavy ion orbiting insisting on the connection with Regge poles and barrier-top resonances. Although the physical content of the phenomenon is well understood semiclassically, a clear signature is hard to be found because the relation between the observation angle and the deflection angle is not one to one.

Key words: G-matrix effective interactions, folding potentials, WKB, Regge poles.

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1. INTRODUCTION

We have a long-term program to understand and describe nucleus-nucleus collisions in terms of one body interaction potential, the optical model potential (OMP). A good understanding of all phenomena occurring in the elastic nucleus-nucleus scattering, which are used typically to extract OMP, and the interpretation of the origin of different aspects, including the well known potential ambiguities, are of crucial importance for finding and justifying the procedures used for predicting nucleus-nucleus OMP in the era of radioactive nuclear beams (RNB), including ours based on double folding [1]. The reliability of these potentials is crucial for the correct description of a number of reactions involving RNBs, from elastic to transfer and breakup, at energies ranging from a few to a few hundred MeV/nucleon. Of particular interest for us is to support the absolute values of the calculated cross sections for reactions used in indirect methods for nuclear astrophysics, see [2] and [3] for the most recent results. In this framework, we treat here the case of heavy ion orbiting, one of the phenomena found over the years to occur in special cases of elastic scattering, well understood semi-classically, but not well documented by specific examples.

The anomalous large-angle scattering of α-particles at moderate energies from elements throughout the periodic table has been a subject of considerable experimental study and has evoked a wide range of novel theoretical explanations [4, 5]. The conventional nuclear optical potential can explain much, if not all, of the anomalous scattering. The dominant physical parameter determining back-angle scattering is the strength, W, of the imaginary part of the optical potential. Lowering of W by a modest factor of two or three lead to changes in back-angle scattering by several orders of magnitude. This effect was dubbed in literature improperly as incomplete absorption. This severe sensitivity of back-angle scattering to the imaginary strength of the optical potential was explained as a sudden emergence of the giant resonances of the high-partial-wave strength functions, as W decreases [6]. A more popular explanation is the interference between the wave reflected at the internal angular momentum barrier with the wave reflected at the nuclear radius.

Analysis of several heavy ion elastic scattering angular distribution in the energy range of 4-10 MeV/A conclude that backward-angle structures are caused by very few partial waves close to grazing collision value $\ell = kR$. Consequently, all theoretical approaches have to strengthen the contribution from these partial waves relative to the normal optical or diffraction model. Cowley and Heymann [7] and McVoy [8] parametrize the scattering amplitude by a Regge pole expansion in angular momentum. The explanation in terms of a sequence of Regge poles suggests that the physical mechanism behind the large angle structures could be heavy ion orbiting.

Orbiting could be understood simply in terms of the classical equation of motion. Let a particle m in a strong attractive potential $V(r)$. Then the motion is given by

$$\frac{1}{2}m\dot{r}^2 + \frac{1}{2}\frac{L^2}{mr^2} + V(r) = E \tag{1}$$

Let the effective interaction $U(r,L) = \frac{1}{2}\frac{L^2}{mr^2} + V(r)$ and assume that for a certain angular momentum $L = L_{orb}$ the effective interaction has a maximum U_{max} and $U_{max} = E$. If this condition is satisfied then the radial velocity $\dot{r} = 0$ and the particle is orbiting indefinitely with a radius corresponding to the maximum. For E close to the critical energy the particle remains a finite time in this state.

In this paper we review the semiclassical theory of Brink and Takigawa [9] in relation with heavy ion orbiting, barrier-top resonances and Regge poles. In a second part of the paper we examine the ability of the double folding model of the optical potential to describe orbiting.

2. ORBITING AND REGGE POLES

We start from the radial Schrödinger equation for a real spherical potential

$$-\frac{\hbar^2}{2\mu}\left(\frac{\partial}{\partial r^2} - \frac{\ell(\ell+1)}{r^2}\right)\Psi + V(r)\Psi = E\Psi \tag{2}$$

and assume that the effective potential

$$V_{\text{eff}}(r) = V(r) + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2} \tag{3}$$

has a barrier at a finite radius say $r = r_B$. Then close to the barrier we may write:

$$V_{\text{eff}}(r) \approx V_B - \frac{1}{2}\mu\omega_B^2(r-r_B)^2 \tag{4}$$

$$-\frac{\hbar^2}{2\mu}\frac{\partial^2\Psi}{\partial r^2} - \frac{1}{2}\mu\omega_B^2(r-r_B)^2\Psi = (E-V_B)\Psi \tag{5}$$

In fact a Taylor series expansion of V_{eff} gives:

$$V_{\text{eff}}(r) = V_{\text{eff}}(r_B) + \frac{1}{2}(r-r_B)^2 V''_{\text{eff}}\Big|_{r_B} \tag{6}$$

$$V_B = V_{\text{eff}}(r_B) \tag{7}$$

$$\omega_B = \sqrt{-\frac{V''_{\text{eff}}(r_B)}{\mu}} \tag{8}$$

In Eq.(5) we change the variable

$$x = \sqrt{\frac{\mu\omega_B}{\hbar}}(r-r_B) \tag{9}$$

and Eq.(5) becomes

$$\begin{gathered}\frac{\partial^2\Psi}{\partial x^2} + x^2\Psi + 2\varepsilon\Psi = 0 \\ \text{with} \quad \varepsilon = \tfrac{E-V_B}{\hbar\omega_B}\end{gathered} \tag{10}$$

Friedman and Goebel [10] have shown that resonances (poles in complex energy plane) emerge when

$$\varepsilon_n = -\left(n+\frac{1}{2}\right)i \tag{11}$$

which is just the quantization of the inverted harmonic oscillator well. It results that

$$E_n = V_B - i\left(n+\frac{1}{2}\right)\hbar\omega_B \tag{12}$$

These are poles for fixed angular momentum in complex energy plane. The orbiting angular momentum is defined by

$$V_B(\ell_{\text{orb}}(E)) = E \tag{13}$$

For ℓ close to $\ell_{\rm orb}$ we expand to first order

$$V_B(\ell) = V_B(\ell_{\rm orb}) + \frac{\partial V_B}{\partial \ell}(\ell - \ell_{\rm orb}) = E + \hbar\omega_{\rm orb}(\ell - \ell_{\rm orb}) \tag{14}$$

where

$$\begin{aligned} \hbar\omega_{\rm orb} &= \left.\frac{\partial V_B}{\partial \ell}\right|_{\ell=\ell_{\rm orb}} = \frac{\partial}{\partial \ell}\left[\frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r_B^2}\right]\Bigg|_{\ell=\ell_{\rm orb}} \\ &= \frac{\hbar^2}{2\mu r_B^2}\frac{\partial}{\partial \ell}(\ell^2+l)\Bigg|_{\ell=\ell_{\rm orb}} = \frac{\hbar^2}{2\mu r_B^2}(2\ell_{\rm orb}+1) \end{aligned} \tag{15}$$

The orbiting frequency at the top of the barrier reads :

$$\omega_{\rm orb} = \frac{\hbar}{\mu r_B^2}\left(\ell_{\rm orb} + \frac{1}{2}\right) \tag{16}$$

We can write Eq.(14) in the form

$$E - V_B(\ell) = -\hbar\omega_{\rm orb}(\ell - \ell_{\rm orb}) \tag{17}$$

and continue with Eq.(10)

$$E - V_B(\ell) = \hbar\omega_B\varepsilon = -\hbar\omega_{\rm orb}(\ell - \ell_{\rm orb}) \tag{18}$$

The reduced momentum reads

$$\frac{E - V_B(\ell)}{\hbar\omega_B} = -\frac{\hbar\omega_{\rm orb}}{\hbar\omega_B}(\ell - \ell_{\rm orb}) \equiv -\lambda \tag{19}$$

Therefore we have $\lambda = -\varepsilon$ and the barrier-top resonances translate into Regge poles at

$$\lambda_n = \left(n + \frac{1}{2}\right)i \tag{20}$$

or

$$(\ell_n - \ell_{\rm orb})\frac{\omega_{\rm orb}}{\omega_B} = \left(n + \frac{1}{2}\right)i \tag{21}$$

We get

$$\ell_n = \ell_{\rm orb} + \frac{\omega_B}{\omega_{\rm orb}}\left(n + \frac{1}{2}\right)i \tag{22}$$

or

$$\ell_n = \ell_{\rm orb} + \Gamma_\ell\left(n + \frac{1}{2}\right)i \tag{23}$$

so the lowest pole is exactly at $\ell_0 = \ell_{\rm orb} + i\frac{\Gamma_\ell}{2}$ with $\Gamma_\ell = \frac{\omega_B}{\omega_{\rm orb}}$. Note that Γ_ℓ depends on $\ell_{\rm orb}$ through relation (16)

$$\Gamma_\ell = \frac{\omega_B}{\omega_{\rm orb}} = \frac{\sqrt{-\frac{V''_{\rm eff}(r_B)}{\mu}}}{\frac{\hbar}{\mu r_B^2}\left(\ell_{\rm orb} + \frac{1}{2}\right)} \tag{24}$$

where in Eq.(24) the primes denote the derivative with respect to r_B. We have

$$V_{\text{eff}}(r_B) = V(r_B) + \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r_B^2} \tag{25}$$

$$V'_{\text{eff}}(r_B) = V'(r_B) - \frac{\hbar^2}{2\mu}\ell(\ell+1)\frac{2}{r_B^3} = V'(r_B) - \frac{\hbar^2}{\mu}\frac{\ell(\ell+1)}{r_B^3} \tag{26}$$

$$V''_{\text{eff}}(r_B) = V''(r_B) + \frac{\hbar^2}{\mu}\frac{3\ell(\ell+1)}{r_B^4} \tag{27}$$

In the presence of absorption, Friedman and Goebel [10] conjectured that the pole will be shifted by the quantity

$$\Delta\ell_n = i\frac{\omega(r_B)}{\omega_{\text{orb}}}. \tag{28}$$

3. SEMICLASSICAL ORBITING

We turn now to study the reaction mechanism governing orbiting using semiclassical methods. The far-side dominance observed in some heavy ion elastic scattering angular distributions is not able to explain the behavior of the S-matrix elements at low angular momentum. The reason is of course that the far/near (F/N) decomposition method does not perform a dynamic decomposition of the scattering function, but merely decomposes the scattering amplitude into traveling waves. The intermediate angle structures, have been repeatedly interpreted as arising from the interference of two ranges in angular momenta, $\ell_<$ and $\ell_>$, contributing to the same negative deflection angle. However, the corresponding cross sections, $\sigma_{F<}$ and $\sigma_{F>}$, cannot be isolated because their dynamic content (S-matrix) is not accessible.

The semiclassical uniform approximation for the scattering amplitude of Brink and Takigawa [9] is well adapted to describe situations in which the scattering is controlled by at most three active, isolated, complex turning points. An approximate multireflection series expansion of the scattering function can be obtained, the terms of which have the same simple physical meaning as in the exact Debye expansion for the scattering of light on a spherical well. The major interest in this theory comes from the fact that it can give precious information on the response of a nuclear system to the nuclear interior. An application [11] of this technique helped to clarify the controversial problem of the "Airy oscillation" seen in low energy ^{16}O+^{12}C scattering [13].

We discard the absorptive term in the optical potential and define the effective potential as,

$$V_{\text{eff}}(r) = V(r) + \frac{\hbar^2}{2\mu}\frac{\lambda^2}{r^2}, \;\; \lambda = \ell + \frac{1}{2} \tag{29}$$

where the Langer prescription has been used for the centrifugal term. This guarantees the correct behavior of the semiclassical wave function at the origin [14]. Then we calculate the deflection function,

$$\Theta(\lambda) = \pi - 2\int_{r_1}^{\infty} \frac{\sqrt{\frac{\hbar^2}{2\mu}}\lambda dr}{r^2\sqrt{E_{\text{c.m.}} - V_{\text{eff}}(r)}} \tag{30}$$

where r_1 is the outer zero of the square root, i.e. the radius of closest approach to the scatterer and μ is the reduced mass. Note that with the replacement $\hbar\lambda = b\sqrt{2\mu E}$, Eq.(30) becomes identical with the classical deflection function $\Theta(b)$, where b is the impact parameter. The behavior of $\Theta(\lambda)$ is the one expected for a strong nuclear potential in a *near orbiting* kinematical situation in which the c.m. energy approximately equals that of the top of the barrier for some specific angular momentum. All the measured angular range is classically illuminated. The deflection function exhibit no genuine minima, but rather a pronounced cusp close to an orbiting logarithmic singularity. Therefore any interpretation of structures in angular distributions in terms of Airy oscillations can be discarded. Rather we need an interpretation appropriate for orbiting, a well documented situation in classical physics [15]. We identify the cusp angular momentum as orbiting momentum (λ_{orb}) since this is related to the coalescence of two (barrier) turning points and the innermost turning point given by the centrifugal barrier becomes classically accessible. There are two branches that can be distinguished, an internal branch for low active momenta $\lambda < \lambda_{\text{orb}}$ related to semiclassical trajectories which penetrate into the nuclear pocket and a less developed external (barrier) branch ($\lambda > \lambda_{\text{orb}}$) related to trajectories deflected at the diffuse edge of the potential.

However this simple calculation cannot determine the relative importance of these branches and provides no information about the interference effects of the corresponding semiclassical trajectories. To clarify these points it is best to go into the complex r-plane and look for complex turning points, i.e. the complex roots of the quantity $E_{\text{c.m.}} - V_{\text{eff}} - iW$. This is an intricate numerical problem, because, for a WS optical potential, the turning points are located near the potential singularities and there are an infinite number of such poles. We consider an ideal situation with three, well isolated, turning points for each partial wave.

The multireflection expansion of the scattering function in the Brink-Takigawa approach reads,

$$S_{WKB}(\ell) = \sum_{q=0}^{\infty} S_q(\ell) \tag{31}$$

where,

$$S_0(\ell) = \frac{\exp(2i\delta_1^\ell)}{N(-i\varepsilon)} \tag{32}$$

and for $q \neq 0$,

$$S_q(\ell) = (-)^{q+1} \frac{\exp\left[2i(qS_{32} + S_{21} + \delta_1^\ell)\right]}{N^{q+1}(-i\varepsilon)} \tag{33}$$

In these equations, $\varepsilon = S_{21}/\pi$ and δ_1^ℓ is the WKB (complex) phase-shift corresponding to the turning point r_1, $N(z)$ is the barrier penetrability factor,

$$N(z) = \frac{\sqrt{2\pi}}{\Gamma\left(z + \frac{1}{2}\right)} \exp\left(z \ln z - z\right) \tag{34}$$

and S_{ij} is the action integral calculated between turning points r_i and r_j,

$$S_{ij} = \int_{r_i}^{r_j} dr \left\{ \frac{2\mu}{\hbar^2} [E_{\text{c.m.}} - V_{\text{eff}}(r) - iW(r)] \right\}^{1/2} \tag{35}$$

S_{21} and S_{32} are independent of the integration path provided they lie on the first Riemann sheet and collision with potential poles is avoided. Each term in Eq.(31) has a simple physical interpretation. The first term (the barrier term, denoted also S_B) retains contributions from trajectories reflected at the barrier, not penetrating the internal region. The q-th term corresponds to trajectories refracted q times in the nuclear interior with $q-1$ reflections at the barrier turning point r_2. Summation of terms $q \geq 1$ can be recast into a single term,

$$S_I = \frac{\exp\left[2i(S_{32} + S_{21} + \delta_1^\ell)\right]}{N(-iS_{21}/\pi)^2} \frac{1}{1 + \exp\left[2iS_{32}\right]/N(-iS_{21}/\pi)} \tag{36}$$

and is known as the internal barrier scattering function. When the absorption in the nuclear interior is large, the second factor in the above equation reduces to one and we are left with the expression used in [16]. Since the semiclassical scattering function is decomposed additively, $S_{\text{WKB}} = S_B + S_I$, the corresponding total scattering amplitude is decomposed likewise as $f_{\text{WKB}} = f_B + f_I$ and conveniently the corresponding barrier and internal barrier angular distributions are calculated as $\sigma_{B,I} = |f_{B,I}|^2$, using the usual angular momentum expansion of the amplitudes.

The accuracy of the semiclassical calculation is usually checked by comparing the barrier and internal barrier absorption profiles with the exact quantum-mechanical result. When the action integrals are calculated accurately, the semiclassical B/I expansion is an $exact$ decomposition of the quantum result. The internal component gets significant values up to the grazing angular momentum ℓ_g and is negligibly small beyond this value. The barrier component resembles a strong absorption profile and this justifies the interpretation that it corresponds to that part of the flux not penetrating into the nuclear interior. For values near the orbiting angular momentum $\ell_{\rm orb}$, the two components interfere and a downward spike appears in the total profile, in complete agreement with the quantum result. This is the famous Grühn-Wall spike [17] introduced phenomenologically to explain ALAS for α-particle scattering, and appears here as a strong interference between barrier and internal barrier amplitudes. Second, the B/I components are almost decoupled in the angular momentum space and therefore they will contribute in different angular ranges.

A better insight into this technique is obtained by further decomposing the B/I components into far and near (BF/BN and IF/IN) subcomponents. The barrier component dominates the forward angle region. Fraunhofer diffractive oscillations appear as the result of BF and BN interference. At large angles, the internal contribution accounts for the full cross section.

Thus, the intermediate angle exotic structure in angular distributions for the elastic scattering of ^{6}Li on ^{16}O [18] can be understood as a result of coherent interference of two far-side subamplitudes generated by different terms in the uniform multireflection expansion of the scattering function (terms q=0 and q=1 in Eq.(31)), corresponding to the scattering at the barrier and the internal barrier. This interference effect appears as a signature of a surprisingly transparent interaction potential for loosely bound nucleus ^{6}Li at this low energy which allows part of the incident flux to penetrate the nuclear interior and reemerge with significant probability.

The multireflection series (31) is the uniform approximation analogue of the Debye expansion of the scattering function. Anni [11] used Eq.(31) to interpret ^{16}O+^{12}C scattering data at 132 MeV assuming a surface transparent optical potentials and convincingly showed that medium angle structures are given by interference effects of the barrier/internal amplitudes and more precisely by interference between saddles appearing in the first and second term of multireflection expansion and therefore cannot be interpreted as a manifestation as a nuclear rainbow and associated Airy oscillation.
In the following we will provide a third interpretation by using explicitly the orbiting conditions and explicitly calculating Sommerfeld poles near the real axis.

We will made a totally different assumption on the physical nature of the phenomenon: the absorption is negligible near the barrier but strong in the nuclear interior. In such conditions, the scattering amplitude is described by the barrier component alone modified slightly by the barrier penetration factor :

$$S_n \approx \frac{e^{2i\delta_1(\lambda)}}{N(-i\frac{S_{21}}{\pi})} \tag{37}$$

which is slightly changed compared with the original Brink-Takigawa formulation. Since δ_1 describes trajectories reflected at outer turning point, the scattering amplitude $S \sim e^{2i\delta_1}$ will be very much similar to that given by the strong absorption model. The action S_{21} is given by

$$S_{21} = \int_{r_2}^{r_1} \left[\frac{2\mu}{\hbar^2}(E - V_{\text{eff}}(r))\right]^{\frac{1}{2}} dr \tag{38}$$

with

$$V_{\text{eff}}(r) = V(r) + V_c(r) + \frac{\hbar^2}{2\mu}\frac{\lambda^2}{r^2} \quad , \quad \lambda = \ell + \frac{1}{2} \tag{39}$$

where V_c is the Coulomb potential. Near the barrier, the absorption is small and the effective potential is almost real and

$$V_{\text{eff}}(r) = V_B + \frac{1}{2}\mu\omega_B^2(r - r_B)^2 \tag{40}$$

with

$$\omega_B = \sqrt{-\frac{V''_{\text{eff}}(r)}{\mu}}\Bigg|_{r=r_B} \tag{41}$$

where in Eq.(41) the prime denote the derivative with respect to r. Eq.(38) becomes:

$$S_{21} = \int_{r_2}^{r_1} \left[\frac{2\mu}{\hbar^2}(E - V_B - \frac{1}{2}\mu\omega_B^2(r - r_B)^2\right]^{\frac{1}{2}} dr \tag{42}$$

With the variable change,

$$r - r_B = \frac{\hbar x}{\mu\omega_B}, \quad x = \frac{\mu\omega_B}{\hbar}(r - r_B), \quad dx = \frac{\mu\omega_B}{\hbar}dr \tag{43}$$

$$S_{21} = \frac{\hbar}{\mu\omega_B}\int_{x_2}^{x_1} \left[\frac{2\mu}{\hbar^2}(E - V_B) - x^2\right]^{\frac{1}{2}} dx \tag{44}$$

$$x_2 = \frac{\mu\omega_B}{\hbar}(r_2 - r_B), \qquad x_1 = \frac{\mu\omega_B}{\hbar}(r_1 - r_B) \tag{45}$$

the action integral S_{21} can be calculated exactly as,

$$S_{21} = \pi\frac{E - V_B}{\hbar\omega_B} \tag{46}$$

So that ε entering Eq.(32) is

$$\varepsilon = \frac{E - V_B}{\hbar\omega_B} \tag{47}$$

Eqs.(32) and (34) show that the poles in S-matrix are given by the poles of the Gamma function [12]. Taking $z = -i\varepsilon$ in Eq.(34)

$$\frac{1}{2} - i\varepsilon_n = -n \tag{48}$$

$$\varepsilon_n = -i\left(n + \frac{1}{2}\right) \tag{49}$$

$$\frac{E_n - V_B}{\hbar\omega} = -i\left(n + \frac{1}{2}\right) \tag{50}$$

$$E_n = V_B - i\hbar\omega_B\left(n + \frac{1}{2}\right) \tag{51}$$

Eq.(51) represents the Bohr-Sommerfeld quantization for the inverted oscillator well. These are precisely the barrier-top resonances (Regge poles) of Friedman and Goebel [10].
Now we use the orbiting condition

$$V_B\left(\ell_{\mathrm{orb}(E)}\right) = E \tag{52}$$

to obtain poles in angular momentum. For ℓ close to orbiting momentum we expand the potential to first order

$$V_B(\ell) = V_B(\ell_{\mathrm{orb}}) + \frac{\partial V_B}{\partial \ell}(\ell - \ell_{\mathrm{orb}}) \equiv E + \hbar\omega_{\mathrm{orb}}(\ell - \ell_{\mathrm{orb}}) \tag{53}$$

where the orbiting frequency is

$$\hbar\omega_{\mathrm{orb}} = \left.\frac{\partial V_B}{\partial \ell}\right|_{\ell_{\mathrm{orb}}} = \frac{\hbar^2}{\mu r_B^2}\left(\ell_{\mathrm{orb}} + \frac{1}{2}\right) \tag{54}$$

Taking in (53) $E = E_n$ and $\ell = \ell_n$ and then combining with (51) we obtain

$$\left\{ \begin{array}{lcl} V_B - E_n & = & \hbar\omega_{\mathrm{orb}}(\ell_n - \ell_{\mathrm{orb}}) \\ V_B - E_n & = & i\hbar\omega_B\left(n + \frac{1}{2}\right) \end{array} \right. \tag{55}$$

We get

$$\hbar\omega_{\mathrm{orb}}(\ell_n - \ell_{\mathrm{orb}}) = i\hbar\omega_B\left(n + \frac{1}{2}\right) \tag{56}$$

so on

$$\ell_n = \ell_{\mathrm{orb}} + i\frac{\omega_B}{\omega_{\mathrm{orb}}}\left(n + \frac{1}{2}\right) \tag{57}$$

These are the barrier-top poles in angular momentum space. Eq.(57) shows that the nearest pole to real axis has a real part given precisely by the orbiting momentum and a width:

$$\Gamma_\ell = \frac{\omega_B}{\omega_{\mathrm{orb}}} \Rightarrow \ell_n = \ell_{\mathrm{orb}} + i\Gamma_\ell\left(n + \frac{1}{2}\right) \tag{58}$$

Going back to the amplitude (37), the phase shift reads :

$$2i\widetilde{\delta_1}(\lambda) = 2i\delta_1(\lambda) + \ln\left[\Gamma\left(\frac{1}{2} - i\varepsilon\right)\right] - \frac{1}{2}\ln(2\pi) + (z - z\ln z)|_{z=-i\varepsilon} \tag{59}$$

Taking into account the equations (47,55) or (18) we have

$$\varepsilon_n = -\frac{\hbar\omega_{\rm orb}}{\hbar\omega_B}(\ell_n - \ell_{\rm orb}) = -\frac{\hbar\omega_{\rm orb}}{\hbar\omega_B}(\lambda_n - \lambda_{\rm orb}), \quad \lambda = \ell + \frac{1}{2} \tag{60}$$

At the vicinity of $\lambda = \lambda_{\rm orb}$, where ε_0 is close to zero, the equation (59) is separated in a smooth part labeled $g(\lambda)$ and a part including the logarithmic singularity, namely:

$$2\widetilde{\delta_1}(\lambda) = g(\lambda) + (\varepsilon_0 \ln(-i\varepsilon_0) - \varepsilon_0) \tag{61}$$

Here ln denotes the principal determination of the complex logarithm. We have

$$\begin{aligned} \ln(-i\varepsilon_0) &= \ln|\varepsilon_0| - i\frac{\pi}{2} \qquad \varepsilon_0 > 0 \\ \ln(-i\varepsilon_0) &= \ln|\varepsilon_0| + i\frac{\pi}{2} \qquad \varepsilon_0 < 0 \end{aligned}$$

which implies that the imaginary part of $\varepsilon_0 \ln(-i\varepsilon_0) - \varepsilon_0$ has the same sign regardless of whether ε_0 is positive or negative. This smooth part of the singularity, namely $-i\frac{\pi}{2}|\lambda - \lambda_{\rm orb}|$, is included in the function g and we are left with :

$$2\widetilde{\delta_1}(\lambda) = g(\lambda) + (\varepsilon_0 \ln(|\varepsilon_0|) - \varepsilon_0) \tag{62}$$

Using Eq.(62) and the fact that

$$\frac{d}{d\varepsilon_0}(\varepsilon_0 \ln(|\varepsilon_0|) - \varepsilon_0) = \ln(|\varepsilon_0|) \tag{63}$$

we obtain the following semiclassical deflection function

$$\Theta(\lambda) = 2\widetilde{\delta}_1'(\lambda) = g'(\lambda) + \frac{\omega_{\rm orb}}{\omega_B}\ln\left[\frac{\omega_{\rm orb}}{\omega_B}|\lambda - \lambda_{\rm orb}|\right] \tag{64}$$

(the prime being taken with respect to λ) which displays the normal logarithmic singularity near the orbiting on angular momentum. Thus the main signature of the heavy ion orbiting will be a logarithmic singularity in the semiclassical deflection function.

The Equation (64) is valid for every $\lambda - \lambda_{\rm orb}$ positive or negative. Note that we have neglected the weak dependence of orbiting frequency on angular momentum (54).

4. REGGE POLES

A long standing problem in the α-nucleus scattering at energies above the Coulomb barrier is the so called ALAS, a strong increase of the cross section at large angles. It was observed by Grühn and Wall [17] that a downward narrow spike superimposed on the smooth-cut-off model for ℓ values near grazing $\ell = kR$ aided materially to explain ALAS. The Grühn and Wall dip is explained semiclassically as a strong destructive interference between the internal barrier and barrier components of the scattering amplitude near the orbiting momentum. Alternatively, the dip is explained as an interference pole-background components of the scattering amplitude for highly transparent potentials, such that the low absorption is not able to suppress the resonant effects in the low partial waves. Semiclassically, these effects appear as a consequence of multiple reflections of the internal amplitude between the most internal complex turning points of the potential. In fact a common property of the WS potentials which describe well the data, is that they possess several narrow shape (molecular) resonances located in the most active waves. In this section we examine this effect in terms of a purely phenomenological Regge pole approximation.

For this purpose we adopt the "product" representation of the S-matrix [8],

$$S(\ell) = S_{bkg}(\ell) S_{pole}(\ell) \tag{65}$$

where the background (bkg) component is borrowed from the strong absorption model of Ericson [19],

$$S_{bkg} = \left[1 + \beta \exp(-i\alpha) \exp\left(\frac{L-l}{\Delta}\right)\right]^{-1} \tag{66}$$

We note that an alternative description in terms of additive pole-background components is possible [7]. For the pole term we adopt the expression,

$$S_{pole}(\ell) = \prod_{j=1}^{2} \left[1 + i \frac{D_j(\ell)}{l - L_j - i\hat{\Gamma}_j(\ell)/2}\right] \tag{67}$$

This term describes resonances in ℓ centered at L_j with total width $\hat{\Gamma}_j$. In line with McVoy [8] we assume the zeros and the widths slowly ℓ dependent and vanishing exponentially as $\ell \to \infty$,

$$D_j(\ell) = \frac{D_j}{1 + \exp(\frac{l-L}{\Delta_j})} \tag{68}$$

$$\hat{\Gamma}_j(\ell) = \frac{\Gamma_j}{1 + \exp(\frac{l-L}{\Delta_j})} \tag{69}$$

Clearly, D measures the distance between the pole ($p = 1/2\Gamma$) and the zero ($z = 1/2\Gamma - D$). The model has 12 parameters, twice as much as the WS model. The reason is that we were not able to find a single pole unitary solution for both background and pole components. Since the problem is highly nonlinear there is no guarantee for the uniqueness of the solution. We used a Monte Carlo procedure to generate input parameters and then minimized the usual χ^2 objective function.

5. CONCLUSIONS

We have reviewed the semiclassical theory of Brink and Takigawa [9] in connection with heavy ion orbiting. The phenomenon is quite frequent for α-particle scattering on light targets in the energy range 5-10 MeV/A where significant increase in the cross section at large angles is found. A possible signature will be to find a logarithmic singularity is the semiclassical deflection function associate with a downward spike in the absorption profile near the grazing angular momentum $\ell = kR$.

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