MACROSCOPIC ENERGY OF DEFORMATION FOR BINARY FUSION SHAPES

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1 Introduction

Coulomb energy formula for uniform charge distribution has been introduced in [1]. The nuclear energy accounting for finite range of nuclear forces has been expressed by a potential given in terms of a double volume integral of a Yukawa-plus-exponential folding function in [2]. Both formulae apply to nuclear shapes with constant charge density within the volume. Fot the case of two nuclei system as in fusion or fission processes, charge density vary from the two separated nuclei to the compound nucleus value. A simple law of variation and its dependence on different values of the shape parameters is derived in this work. The influence of the charge density variation on the Coulomb, nuclear and total macroscopic energy term is presented for a two-center nuclear system.

2 Charge density variation

A typical nuclear configuration for fusion phenomena is described by two intersected ellipsoids with (a_1, b_1) and (a_2, b_2) semiaxes, separated by a plane $z = z_s$ as in Fig.1. The two volumes are defined by the shape parameters. We will refer furtheron only to the left side of the shape corresponding to the heavy fragment (A_1, Z_1), the demonstration for the light fragment being similar. The right side volume of this intermediary shape is:

$$V_1 = \pi \left(\frac{2}{3} a_1 - \frac{z_s}{3a_1^2} + z_s \right) b_1^2 \qquad (1)$$

The whole ellipsoid volume for the (a_1, b_1) shape parameters is:

$$V_{10} = \frac{4}{3}\pi a_1 b_1^2 = \frac{4}{3}\pi R_{1x}^3 = \frac{4}{3}\pi r_0^3 A_{1x} \tag{2}$$

where R_{1x} is the radius of the sphere having the same volume and A_{1x} is the corresponding mass number. We consider the ellipsoidal shape (a_1, b_1, z_s) as having the atomic properties as it were a whole nucleus (A_{1x}, Z_{1x}); that means the charge density of the shape is determined by its geometric correspondence to (A_{1x}, Z_{1x}); thus Z_{1x} is the atomic number if the heavy fragment is a complete ellipsoid with (a_1, b_1) semiaxes. Variation of Z_{1x}/A_{1x} must also comply to their initial and final values:

$$(\frac{Z_{1x}}{A_{1x}})_f = \frac{Z_0}{A_0} \tag{3}$$

where Z_0 and A_0 are the final values corresponding to the compound nucleus and:

$$(\frac{Z_{1x}}{A_{1x}})_i = \frac{Z_1}{A_1} \tag{4}$$

where Z_1 and A_1 are the initial values of the target nucleus. A variation law fulfilling these conditions is:

$$\frac{Z_{1x}}{A_{1x}} = \frac{1}{A_0 - A_1}\left[(A_{1x} - A_1)\frac{Z_0}{A_0} + (A_0 - A_{1x})\frac{Z_1}{A_1}\right] \tag{5}$$

For the surface coefficients c_{s1i} and c_{s2i}, with the general expression:

$$c_s = a_s(1 - \kappa I^2) \tag{6}$$

and the intermediary isospin values:

$$\begin{aligned} I_{1i} &= \frac{N_{1i} - Z_{1i}}{A_{1i}} \\ I_{2i} &= \frac{N_{2i} - Z_{2i}}{A_{2i}} \end{aligned} \tag{7}$$

Obviously, $Z_{1i} + Z_{2i} = Z_0$ and $N_{1i} + N_{2i} = N_0$. Hence the intermediary surface coefficients are:

$$\begin{aligned} c_{s1i} &= a_s(1 - \kappa I_{1i}^2) \\ c_{s2i} &= a_s(1 - \kappa I_{2i}^2) \end{aligned} \tag{8}$$

For the same fusion reaction, an ellipsoidal projectile can change its shape parameters in different ways along the overlapping region: it can preserve its initial b_{2f} semiaxis or b_2 can become larger up to the limit where $b_2 = b_0$, the semiaxis of the compound nucleus. Between these two limits, b_2 can take any values, provided that the volume V_2 does not become larger then its initial value.

3 Coulomb and nuclear Yukawa+exponential energies

We use the published formulae of E_C [1] and E_Y [2], for the left and right shape energy values. The Coulomb term reads:

$$E_C = \frac{2\pi}{3}\rho_e \int_{z_{min}}^{z_{max}} dz \int_{z_{min}}^{z_{max}} dz\prime \cdot F_C(z, z\prime) \tag{9}$$

where:

$$\begin{aligned} F_C(z,z\prime) = &\\ &\left\{\rho(z)\rho(z\prime)\frac{K(k)-2D(k)}{3}\right.\\ &\left[2(\rho^2(z)+\rho^2(z\prime)-(z-z\prime)^2+1.5(z-z\prime)\left(\frac{d\rho^2(z\prime)}{dz\prime}-\frac{d\rho^2(z)}{dz}\right)\right]+\\ &K(k)\left\{\frac{\rho^2(z)\rho^2(z\prime)}{3}+\left[\rho^2(z)-0.5(z-z\prime)\frac{d\rho^2(z)}{dz}\right]\left[\rho^2(z\prime)+0.5(z-z\prime)\frac{d\rho^2(z\prime)}{dz\prime}\right]\right\}\Bigg\}\\ &\frac{1}{[(\rho(z)+\rho(z\prime))^2+(z-z\prime)^2]^{1/2}} \end{aligned} \tag{10}$$

where

$$\begin{aligned} k^2 &= \frac{4\rho(z)\rho(z\prime)}{[\rho(z)+\rho(z\prime)]^2+(z-z\prime)^2}\\ D(k) &= \frac{K(k)-K\prime(k)}{k^2} \end{aligned} \tag{11}$$

If $z = z\prime$ we have:

$$F(z, z\prime) = \frac{4\rho^3(z)}{3} \tag{12}$$

For our two intersected nuclei system shape, the Coulomb energy can be written as [3]:

$$E_C = \frac{2\pi}{3}(\rho_{e1}^2 F_{C1} + \rho_{e2}^2 F_{C2} + 2\rho_{e1}\rho_{e2} F_{C12}) \tag{13}$$

The Yukawa-plus-exponential energy term reads:

$$E_Y = \frac{1}{4\pi r_0^2}[c_{s1} F_{EY1} + c_{s2} F_{EY2} + 2(c_{s1} c_{s2})^{1/2} F_{EY12}] \tag{14}$$

where [2]:

$$F_{EYi} = \int_0^{2\pi} \int_{-a_i}^{z_s} \int_{-a_i}^{z_s} F_{Yi}^{(1)} F_{Y2}^{(i)} Q^{(i)} d\phi dz dz\prime \tag{15}$$

where $FY1, 2$ and $Q(i)$ are dependent on the binary shape geometry.

The total macroscopic deformation energy is

$$E_b^{macro} = (E_C - E_C^{(0)}) + (E_Y - E_Y^{(0)}) \tag{16}$$

4 Results

Calculations have been performed for four laws of variation of the argon radius from its initial value R_{2f} up to the final one R_{2i}, when the projectile is totaly embeded in the compound nucleus. Fig. 1 represents the four cases of shape sequence. Since beyond the touching point the three energies are identical, calculations are presented only up to R_n=1. Differences are more significant in the last part of the fusion process. Higher E_C and $E_b^{(macro)}$ values correspond to the situation when the projectile enters the target with initial radius. The bump at R_n=0.2 appears because of the higher charge density for the shape preserving the initial radius projectile against an increased radius. When the light nucleus changes its shape so as to have enlarged radius, close to the compound nucleus semiaxis, $E_b^{(macro)}$ decreases with about 7 MeV. Last one presents the situation when $R_{2i} = b_{2f}$. Here the three energies E_C, E_Y and $E_b^{(macro)}$ are drawn for the four different entrance paths. E_C increases with 2-3 MeV from $R_{2i} = b_0$ to $R_{2i} = b_{2f}$, when projectile nucleus is not geometrically modified. Both E_C and E_Y have higher values for $R_{2i} = b_{2f}$. One can see on the $E_b^{(macro)}$ plot that the curve constantly lowers as R_{2i} becomes larger up to b_0.

The studied case is the synthesis of a superheavy nucleus by the reaction ^{54}Cr+^{238}U $\rightarrow^{292}$116. Both target and projectile nuclei are deformed

($\beta_2^{(^{54}Cr)}$=0.180, $\beta_2^{(^{238}U)}$=0.215). $^{292}116$ is considered spherical. Differences are obvious. Due to the increase of b_{2i}, charge density becomes more diffuse and E_C decreases. Fig 9 shows that E_Y is also lower for a path with $b_{2i} = b_{20}$. As a consequence, $E_b^{(macro)}$ differences reach about 7 MeV, favouring configurations with projectile geometry appoaching more the synthesized shape.

5 Conclusions

Charge density is introduced as a shape dependent quantity. Its influence manifest both on the Coulomb and nuclear Yukawa-plus-exponential term. The result of calculations on fusion reactions shows a lower macroscopic barrier for configurations where projectile shape approach the synthesized nucleus dimensions as overlapping occurs. This is due to the diffusing and lowering charge density from the target value to the final nucleus. Differences can reach about 3-4 MeV for light nuclei reactions and up to 10 MeV for heavy ions toward superheavy synthesis reactions.

References

[1] K. T. Davies, A. J. Sierk, J. Comp. Phys. **18**,311(1975).

[2] H. J. Krappe, J. R. Nix, A. J. Sierk, Phys. Rev. **C20**,992(1979).

[3] D. N. Poenaru, M. Ivascu, D. Mazilu, Comp. Phys. Com. **19**,205(1980).

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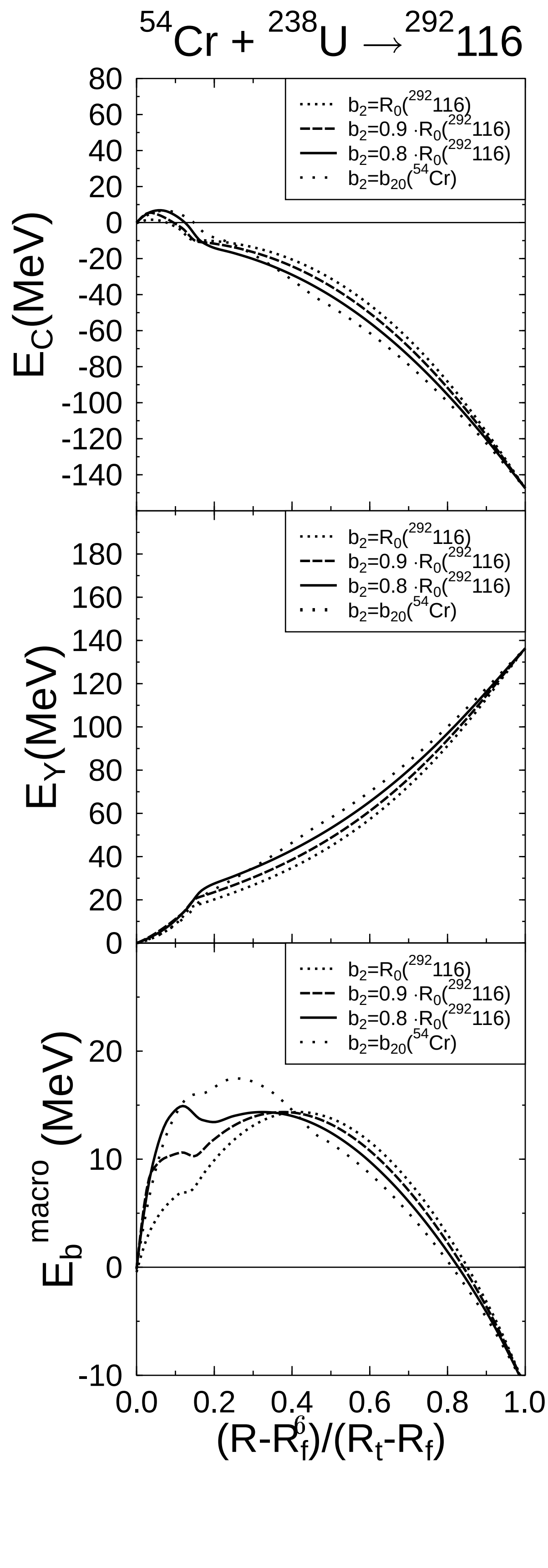
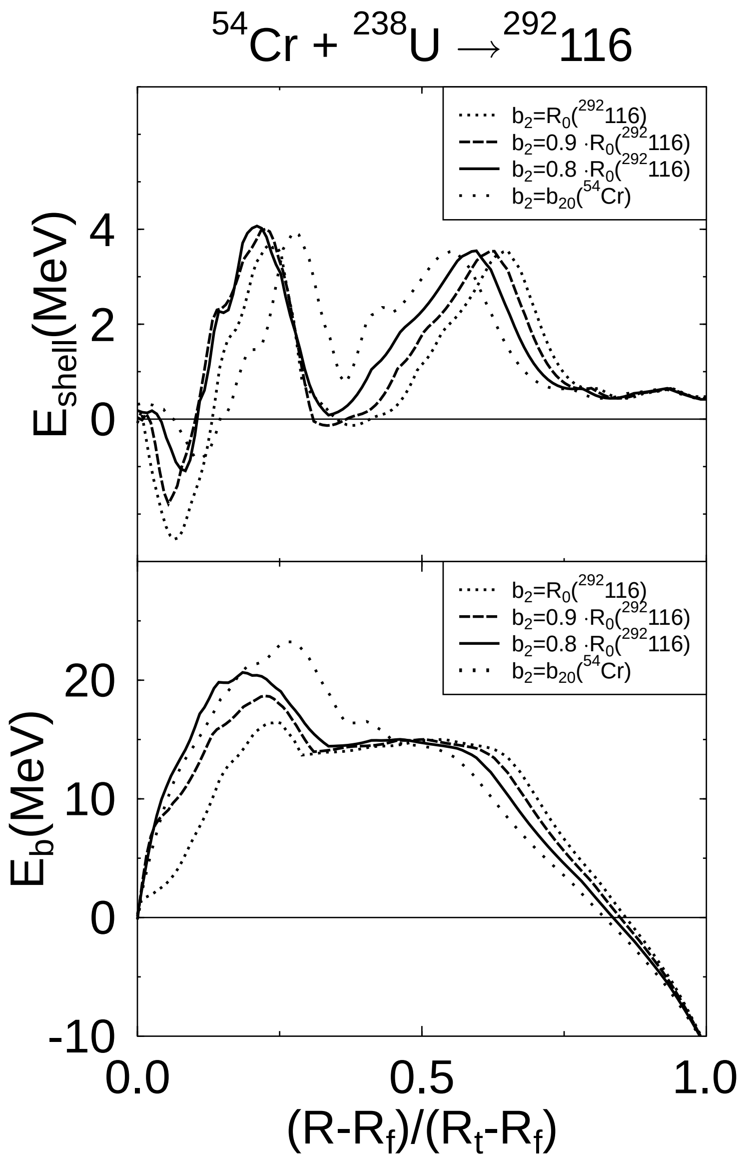
Figure captions

Fig. 1. Coulomb E_C nuclear E_Y and macroscopic barrier E_b^{macro} for the four geometric paths of the projectile nucleus leading to the synthesis of superheavy nucleus $^{292}116$.

Fig. 2. Quantitative comparaison of E_C, E_Y and macroscopic barriers E_b^{macro} for the synthesis of superheavy nucleus $^{292}116$.



54Cr + 238U → 292116
b2=R0(292116)
b2=0.9 ·R0(292116)
b2=0.8 ·R0(292116)
b2=b20(54Cr)
EC(MeV)
EY(MeV)
Eb macro (MeV)
(R-Rf)/(Rt-Rf)



$^{54}Cr + ^{238}U \rightarrow ^{292}116$
$b_2=R_0(^{292}116)$
$b_2=0.9 \cdot R_0(^{292}116)$
$b_2=0.8 \cdot R_0(^{292}116)$
$b_2=b_{20}(^{54}Cr)$
E_{shell}(MeV)
4
2
0
$b_2=R_0(^{292}116)$
$b_2=0.9 \cdot R_0(^{292}116)$
$b_2=0.8 \cdot R_0(^{292}116)$
$b_2=b_{20}(^{54}Cr)$
E_b(MeV)
20
10
0
-10
0.0
0.5
1.0
$(R-R_f)/(R_t-R_f)$

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